

**Mu Alpha Theta 2002 National Competition
Theta Individual Test**

- 1) How many 5 digit palindromes are there? (Palindromes are numbers which read the same forwards and backwards).
- A) 1000 B) 100 C) 2187 D) 900 E) NOTA
- 2) Find the remainder when $x^5 + 4x^4 - 3x^2 + 1$ is divided by $x^2 - 1$.
- A) $x - 1$ B) -1 C) $3x$ D) x E) NOTA
- 3) To which of the following is $\frac{(x + y)^3(z^2 - 1)}{(x^3 + y^3)(z^4 - 1)(x + y + z)}$ equivalent?
- A) $\frac{x^2 y^2}{z^2 + 1}$ B) $\frac{x^2 + y^2}{z^2 + 1}$ C) $((x^2 - xy + y^2)(z^2 + 1)(x + y + z))^{-1}$
- D) $\frac{x^2 + y^2}{(x + y + z)^2}$ E) NOTA
- 4) What is the sum of the roots of $x^3 - x^2 - 4x - 6 = 0$.
- A) 2 B) -1 C) 3 D) -3 E) NOTA
- 5) What is the absolute value of the difference between 105311^2 and 105305^2 .
- A) 1263696 B) 1243692 C) 210622 D) 210610 E) NOTA
- 6) Find the sum of the values for the solutions for x, y, and z:
- $$\begin{aligned}x + 2y + 6z &= 10 \\ 3x + 4y + 2z &= 12 \\ 2x + 3y + 3z &= 6\end{aligned}$$
- A) 2 B) 16 C) 8 D) -4 E) NOTA
- 7) How many rectangles can be drawn on an 8x8 chess board which have odd area?
- A) 100 B) 200 C) 300 D) 400 E) NOTA

- 8) Evaluate $\sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}$.
- A) 2 B) $2\sqrt{2}$ C) $\sqrt{3}$ D) $2\sqrt{3}$ E) NOTA
- 9) What is the minimum distance from (2,3) to a point on the line $y = -x + 8$.
- A) $\sqrt{2}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{2\sqrt{3}}{3}$ D) $\frac{\sqrt{3}}{3}$ E) NOTA
- 10) A ball is dropped from a height of 100 feet onto a hard level surface. Suppose that each time it bounces, it rebounds to half of its previous height. If the ball continues to bounce indefinitely, find the total distance that it travels.
- A) 200 ft B) 300 ft C) 150 feet D) 400 ft E) NOTA
- 11) Suppose a sequence is defined as follows. $a_1 = 2$, $a_{n+1} = \frac{1}{3 - a_n}$
- It can be shown that the sequence is decreasing and $0 \leq a_n \leq 2$ for all n . Find the limit of the sequence.
- A) $\frac{3 + \sqrt{5}}{2}$ B) $\frac{1}{3}$ C) $\frac{3 - \sqrt{5}}{2}$ D) 1 E) NOTA
- 12) Two marbles are drawn from a vase which contains 4 white marbles, 6 blue marbles, and 8 green marbles. What are the odds that if I pick two marbles, they will be the same color?
- A) $\frac{49}{153}$ B) $\frac{58}{153}$ C) $\frac{15}{153}$ D) $\frac{53}{156}$ E) NOTA
- 13) How many six digit numbers (leading digit cannot be zero) are there such that any two adjacent digits have a difference of no more than one? Consider that 0 can be adjacent to 9.
- A) 2187 B) 2430 C) 320 D) 288 E) NOTA
- 14) A circle is inscribed in a square which is inscribed in another circle. What is the ratio of the area of the inner circle to the outer circle?
- A) 1:2 B) 1:3 C) 1:4 D) $\sqrt{2} : 2$ E) NOTA

- 15) Find the sum of all real x such that $|x^2 - 3| = |3x + 1|$.
- A) -2 B) -1 C) 0 D) 3 E) NOTA
- 16) If $f(x) = x^3 + 3x - 5$ and $f(g(x)) = x^5 - 3x^4 + x^2 - 1$ then what is $g(2)$?
- A) -2 B) -1 C) 0 D) 1 E) NOTA
- 17) How many positive integers are there whose digits are all different?
- A) 623,529 B) 9,864,100 C) 986,409 D) 8,877,690
E) NOTA
- 18) Suppose a bookstore purchases 10 copies of a book at \$6.00 each and sells them at \$12.00 each with the understanding that at the end of a 3-month period any unsold copies can be redeemed for \$2.00 each. Find the net revenue of the bookstore owner at the end of the three-month period.
- A) $R(x) = 10x - 40$ B) $R(x) = 12x - 60$ C) $R(x) = 10x + 20$
D) $R(x) = 12x - 2$ E) NOTA
- 19) 1024 people participate in a chess tournament in which each player plays all the others exactly once. How many games are played?
- A) 523776 B) 575986 C) 361840 D) 610053 E) NOTA
- 20) Three married couples, one of which is Ellen and Michael Pearson, have purchased theatre seats and are seated in a row consisting of just six seats. If they sit completely randomly, what is the probability that Ellen and Michael are seated next to each other?
- A) $\frac{2}{3}$ B) $\frac{1}{15}$ C) $\frac{1}{3}$ D) $\frac{1}{5}$ E) NOTA
- 21) The wages of three men for four weeks is \$108. At the same rate of pay, how many weeks will 5 men work for \$135?
- A) 2.5 B) 3 C) 3.5 D) 4 E) 4.5

- 22) Two boys paddled a canoe 6 miles down a river to the point where the river flowed into a lake; then they paddled 4 miles across the lake to a fishing ground. Later they returned over the same route to their starting point. If the trip to the fishing ground required 2 hours, the return trip required 4 hours, and there was no current in the lake, find the rate of the current (mph) in the river?
- A) $5/4$ B) 2 C) $5/2$ D) 4 E) NOTA
23. If $x^3 + cx + d = 0$ has a pair of equal roots, find $4c^3 + 27d^2$.
- A) -5 B) 0 C) 23 D) 31 E) NOTA
24. Solve the inequality: $\left| \begin{matrix} x & x-2 \\ 5 & 10 \end{matrix} \right| < 0$
- A) $x < -3$ B) $x < -2$ C) $x > 2$ D) $x > 3$ E) NOTA
25. ABCD is a trapezoid with AB the lower base. Angle A measures 90 degrees, Angle B measures 60 degrees, and $BC = CD = 8$. Find the area of the trapezoid.
- A) 20 B) $20\sqrt{3}$ C) 40 D) $40\sqrt{3}$ E) 60
26. Find the measure of an angle that is equal to $4/5$ of its supplement.
- A) 45 degrees B) 60 degrees C) 75 degrees D) 80 degrees
E) NOTA
27. Compute the value of x if $5x - 1 = 3 + \frac{2}{1 + \frac{2}{1 + \frac{2}{\dots}}}$
- A) 1 B) -2, 1 C) $2/5$ D) $6/5$ E) NOTA
28. The mass of a spherical body varies jointly as its density and the cube of its radius. Find the ratio of the mass of Jupiter to that of Earth if the density of Jupiter is $5/22$ that of Earth and its radius is 11 times that of Earth.
- A) $8/125$ B) $5/29282$ C) $5/2$ D) $605/2$ E) NOTA

29. A train moving at r mph can cover a given distance in h hours. By how many mph must its speed be increased to cover the same distance in one hour less time?

A) $\frac{r}{h}$ B) $\frac{r}{h+1}$ C) $\frac{r}{h-1}$ D) $\frac{r+1}{h}$ E) $\frac{r-1}{h}$

30. Let $f(t) = 1 + a^t$. Find $\frac{1}{f(t)} + \frac{1}{f(-t)}$ in lowest terms.

A) 0 B) 1 C) $\frac{2}{1-a^{2t}}$ D) $\frac{2}{1-a^{t^2}}$ E) NOTA

31. A small corporation borrowed \$1,500,000 to expand its product line. Some of the money was borrowed at 8%, some at 9%, some at 10% and the rest at 12%. How much money was borrowed at 10% if the annual interest was \$133,000, the amount borrowed at 8% was four times that borrowed at 12% and the amount at 9% was twice the amount at 10%?

A) \$1,050,000 B) \$125,000 C) \$62,500 D) \$262,500
E) NOTA

- 32) Find c for the quadratic function $f(x) = ax^2 + bx + c$ such that $f(1) = 1$, $f(-3) = 17$ and $f(2) = -1/2$.

A) $\frac{1}{2}$ B) -3 C) $-\frac{3}{2}$ D) $\frac{7}{2}$ E) NOTA

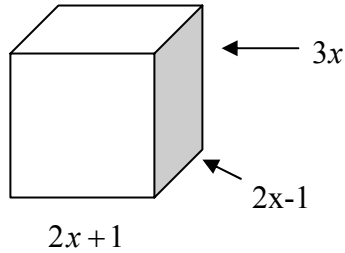
- 33) If $C = 2A - B$ and $A = \begin{bmatrix} 3 & -3 & 2 \\ -1 & 4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 3 & 0 \\ -1 & -4 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 3 & -9 & x \\ -1 & 12 & y \end{bmatrix}$

find $x + y$.

A) -4 B) -2 C) 0 D) 2 E) 4

34) Which expression shows the surface area of the prism pictured ?

- A) $7x - 2$ B) $16x^2 - 1$ C) $32x^2 - 2$ D) $12x^3 - 3x$ E) $24x^3 - 6x$



35) Solve for x : $\sqrt{x-2} = x-4$

- A) 3 B) 6 C) 3 and 6 D) no solution E) 5

36. A line with slope of $\frac{3}{2}$ is given by $3x - 2ky + 2 = 0$. Find k .

- A) 1 B) -1 C) 2 D) -2 E) NOTA

37. A jeep is crossing the desert on a path defined by $y = -\frac{3}{2}x + 1$. A camel is traveling on a path defined by $y = -2x$. Find the coordinate of the point where the paths cross.

- A) (2, -4) B) (-2/7, 4/7) C) (-2, 4) D) the paths do not cross
E) NOTA

38. If $\sqrt{\sqrt{y} + \sqrt{y} + \sqrt{y} + \sqrt{y}} = \sqrt{2}$ then find y .

- A) $\sqrt{2}$ B) $\frac{1}{8}$ C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) NOTA

39) Determine the radius of a circle which passes through the points: (7,1), (6,8) and (-1,7).

- A) 25 B) 3 C) 9 D) 5 E) NOTA

40) Find the value of x such that $4x+6, x+6, 4x-2$ are the first 3 terms of an arithmetic sequence.

- A) $\frac{4}{3}$ B) 0 C) $\frac{8}{3}$ D) 1 E) NOTA

41) The data below is for little sizes of wild rabbits. Calculate the sample mean. (F = frequency and S= size in pounds)

S	1	2	3	4	5	6	7	8	9	10
F	2	4	3	3	3	4	5	3	2	1

- A) 1.83 pounds B) 5.23 pounds C) 8.5 pounds D) 4.25 pounds
E) NOTA

42) Which is NOT equal to 5 ?

- A) $\sqrt{16} + \sqrt{9}$ B) $\frac{\sqrt{125}}{\sqrt{5}}$ C) $\sqrt[3]{125}$
D) $\sqrt[8]{25^4}$ E) $\frac{\sqrt{200}}{\sqrt{8}}$

43. At a fundraiser, cars were washed for \$3.00 each and trucks for \$4.00 each. If \$120 was raised, write an equation which could model the number of cars and trucks that were washed.

I. $3x + 4y = 120$ II. $3x + 4y - 120 = 0$ III. $y = \frac{-3}{4}x + 30$

- A) I only B) II only C) III only D) I and III only E) I, II and III

44) Which is the rule for the function of $\{(-2,5),(-1,4),(3,0),(5,-2)\}$?

- A) $y = x + 3$ B) $y = -x - 3$ C) $y = -x + 3$ D) $y = -3 + x$ E) NOTA

45) A sporting goods store sells a pair of ice skates for 40% more than they cost the store. How much did the skates cost the store if they were sold to a customer for \$70?

- A) \$28 B) \$42 C). \$50 D) \$56 E) NOTA

46) Which point belongs to the graph of the solution set of this system?

$$y > 3$$

$$x - y \geq 3$$

- A). (0,0) B) (6,3) C) (10,4) D) (6, -3) E) NOTA

47. If $f(x) = \frac{2}{3}x - 5$ and $g(x) = 2x^2 + 3x - 5$, find $f(6) + g(-2)$.

- A) -20 B) -4 C) -2 D) 2 E) 10

48. The sum of three consecutive odd integers cannot exceed 36. What is the greatest possible value of the largest integer ?

- A) 9 B) 10 C) 13 D) 14 E. NOTA

49) The line $4x - 7y = 28$ crosses the x-axis at which point?

- A) (7,0) B) (0,7) C). (0, -4) D) (-4,0) E) (-7,0)

50) The point at which the two lines $2x - 5y = 22$ and $15y = 6x - 22$ intersect is :

- A) (11,0) B) (6, -2) C). (16,2) D) They are the same line.
E) They are parallel and do not intersect.

Theta ---Individual Test

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|-------|-------|-------|
| 1. D | 23. B | 45. C |
| 2. E | 24. B | 46. C |
| 3. E | 25. D | 47. B |
| 4. E | 26. D | 48. C |
| 5. A | 27. A | 49. A |
| 6. B | 28. D | 50. E |
| 7. D | 29. C | |
| 8. A | 30. B | |
| 9. E | 31. C | |
| 10. B | 32. D | |
| 11. C | 33. D | |
| 12. A | 34. C | |
| 13. A | 35. B | |
| 14. A | 36. A | |
| 15. C | 37. C | |
| 16. E | 38. D | |
| 17. D | 39. D | |
| 18. A | 40. A | |
| 19. A | 41. B | |
| 20. C | 42. A | |
| 21. B | 43. E | |
| 22. B | 44. C | |

