

2002 National Mu Alpha Theta Convention
Alpha Functions Test

NOTA means "None Of The Above"

Note: In questions 2, 10, 17, and 25, e is the natural base. In question 11, i is $\sqrt{-1}$.

1. Given the function $a(x) = 2x^3 - 3x^2 + 156x - 321$, evaluate $a(-2)$.
A. -5 B. -161 C. -661 D. π E. NOTA

2. Given $b(d) = 2d^2 - 4d + 1$ and $c(d) = 3d - 2$, evaluate $b(c(e))$, rounded to four decimal places.
A. 52.14 B. 12.71 C. 6.155 D. 4.905 E. NOTA

3. The function $c(r) = -\frac{1}{3}r^2 + \frac{9}{4}r - 2$ models the path of an electron as it travels in air, where " r " is time, and " c " is height. What is, to four decimal places, the absolute maximum height the electron achieves (assuming that it can go underground and have a negative height)?
A. 3.375 B. -13.39 C. -2.000 D. 1.797 E. NOTA

4. Let $d(x) = 8x^6 - 5x^5 + 32x^4 - 2x^3 + 24x^2 - x + 40 = 0$. Let A = the sum of the roots of $d(x)$ and B = the product of the roots of $d(x)$. Find the fifth digit *after* (to the right of) the decimal point in the expression $\frac{100A^2\pi}{B!}$.
A. 1 B. 2 C. 3 D. 4 E. NOTA

5. Decompose $\frac{3x-5}{x^2-5x+6}$ into $\frac{2a}{x-2} + \frac{b}{x-3}$. Calculate $-a^2$.
A. $\frac{1}{4}$ B. 2 C. $-\frac{1}{4}$ D. -2 E. NOTA

6. Which of the following are odd functions?
I. $f(x) = |x|$ II. $g(x) = x^3 - x$ III. $h(x) = x^2 + x$ IV. $j(x) = x^5$
A. I only B. II, III C. I, III D. II, IV E. NOTA

7. If $k(x) = \frac{2x-3}{4x+2} + 4$, which of the following is this function's inverse?
A. $k^{-1}(x) = \frac{-4x+2}{2x+3}$ B. $k^{-1}(x) = \frac{-2x-1}{4x-18}$ C. $k^{-1}(x) = \frac{4x+2}{2x-3} + \frac{1}{4}$
D. $k^{-1}(x) = \frac{-2x+5}{4x-18}$ E. NOTA

8. The system
$$\begin{cases} \frac{1}{15}x + \frac{1}{12}y = 1 \\ 0.3x + \frac{4}{15}z = \frac{53}{15} \\ -\frac{1}{7}y + \frac{3}{14}z = -\frac{1}{7} \end{cases}$$
 defines three planes in a three-dimensional

coordinate system. The point where the planes intersect may be labeled (p, q, r) .

Please find $\frac{(4q\sqrt{\pi})(p^2)!(-r^2)}{\left(\sqrt{rqp + pr}\right)^2}!$ rounded to the nearest tenth.

- A. 113.4 B. -113.4 C. A very small negative number (too small for this calculator)
D. A very small positive number (too small for this calculator) E. NOTA

9. Over what interval(s) for a would the function $ax^2 + 5x - 6 = a$ have complex non-real roots?

- A. $\left(-3 - \frac{\sqrt{11}}{2}, -3 + \frac{\sqrt{11}}{2}\right)$ B. $\left(-\infty, -3 - \frac{\sqrt{11}}{2}\right) \cup \left(-3 + \frac{\sqrt{11}}{2}, +\infty\right)$
C. $\left(-\infty, -\frac{25}{24}\right)$ D. $\left[-\frac{25}{24}, +\infty\right)$ E. NOTA

10. Which of the following are one-to-one?

- I. $f(x) = |\log x|$ II. $m(x) = x^2 + 2x - 1$ III. $n(x) = \sqrt{\log_4(x+2)}$ IV. $x = y^2 - 9y$
V. The greatest integer function VI. $o(x) = e^x$ VII. $p(x) = 2^{3x}$
A. I, II, IV, V B. III, VI C. None D. III, VI, VII E. NOTA

11. If $q(x) = i^x$, what is $[q(3)][q(675)][q(444)][q(\frac{1}{2})]$?

- A. $i^{\frac{1}{2}}$ B. $-(-1)^{\frac{1}{4}}$ C. -1 D. 1 E. NOTA

12. Four statements are given below, with a numerical value assigned to each statement in parentheses. Add up the numbers next to the false statements. What is the sum?

- (-4) The area enclosed by $3x^2 + 3y^2 - 24x + 30y + 87 = 0$ is 36π .
(3) The area of $338x^2 + 18y^2 - 2028x + 2700y + 101250 = 0$ is 39π .
(-5) The graph of $18x^2 + 8y^2 - 108x - 32y + 194 = 0$ is a point.
(1) Any three points determine a circle.
A. -4 B. -8 C. -2 D. -3 E. NOTA

13. Let $r(x)$ represent the volume of the solid obtained by revolving the rectangle defined by $(x, 0)$, $(-x, 0)$, $(x, 6)$, and $(-x, 6)$ about the y -axis. Find the value of

$$\frac{r(\pi) + r(-3)}{\frac{2}{5}}. \text{ Simplify as much as possible.}$$

- A. $\frac{3}{5}\pi(\pi^2+9)$ B. $180\pi(\pi-1)$ C. $15\pi(\pi^2+9)$ D. $5\pi(\pi+3)(\pi-3)$ E. NOTA

14. What is the range of $s(g) = \sqrt{\log_3 g}$?

- A. $[0, +\infty)$ B. $(0, +\infty)$ C. $[1, +\infty)$ D. $(1, +\infty)$ E. NOTA

15. Given that $t(x) = \frac{x^2+2}{x-3}$ and $u(x) = \frac{1}{x}$, what is the domain of $t(u(x))$?

- A. $(-\infty, 3) \cup (3, +\infty)$ B. $(-\infty, 0) \cup (0, +\infty)$
 C. $(-\infty, 0) \cup (0, 3) \cup (3, +\infty)$ D. $(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, +\infty)$ E. NOTA

16. If $v(x) = \frac{x^3 - 3x^2 - 3x + 9}{x^2 - 9}$, find the equations of all the asymptotes of $v(x)$.

- A. $x = 3, x = -3$ B. $x = -3, y = x - 3$ C. $x = 3, x = -3, y = 0$ D. $x = 3, y = x + 3$ E. NOTA

17. Given: $w(q) = |q + 3| - 6$, $x(q) = 3q^3 - 5q^2 + 3q - 3$, and $y(q) = 0$. Let a represent the area between the graphs of $w(q)$ and $y(q)$. Let b equal the sum of the reciprocals of the zeros of $x(q)$. Find the correct decimal equivalent, rounded to the nearest integer, of πea^2b .

- A. 44270 B. 11067 C. -44270 D. -11067 E. NOTA

18. $f(y+3) = y^2 + 5y + 6$ and $g(\frac{x}{3}) = 3x^3 - 2x + 5$. Find $[f(y)][g(x)]$.

- A. $3x^2y^2 + 15x^2y + 18x^2 - 2xy^2 - 10xy - 12x + 5y^2 + 25y + 30$
 B. $27x^3y^2 - 27x^3y - 6xy^2 + 5xy + 5y^2 - 5y$
 C. $-81x^3y + 81x^3y^2 - 6xy^2 + 5xy + 5y^2 - 5y$
 D. $27x^2y^2 - 27x^2y - 6xy^2 + 5xy + 5y^2 - 5y$
 E. NOTA

19. Let $y = z(x)$ be a function. Translate $z(x)$ 3 units down, and reflect $z(x)$ over the y -axis. Find an equation for this new function in terms of $z(x)$.

- A. $y = -z(x - 3)$ B. $y = -z(x) - 3$ C. $y = z(-x) - 3$ D. $y = z(-x - 3)$ E. NOTA

20. The equation of a parabola that contains the points $(4, 1)$, $(2, 3)$, and $(-5, 4)$ can be written in the form $x = Ay^2 + By + C$. Find $A + B + C$.

- A. 2 B. 4 C. 6 D. 8 E. NOTA

21. Let $A(x) = (2a + 3b)^x$. Find the eighth term in the binomial expansion of $A(10)$.
 A. $26244a^2b^8$ B. $1180980a^2b^8$ C. $17496a^3b^7$ D. $2099520a^3b^7$ E. NOTA
22. If $B(x)$ is an odd function such that $B(1) = 2$, $B(3) = 5$, and $B(-5) = -1$, then what is the value of $B(B(B(-3)))$?
 A. -5 B. -2 C. 2 D. 5 E. NOTA
23. Which of the following is true concerning functions?
 A. An invertible function's inverse may not necessarily be a function.
 B. An onto and one-to-one function has established a one-to-one correspondence.
 C. A one-to-one function has exactly one intersection point with any horizontal line drawn.
 D. Not all functions are relations
 E. NOTA
24. The product of the zeros of $C(x) = 3x^3 + 10x^2 - 60x - 9$ is:
 A. 3 B. -3 C. 2.88 D. -10/3 E. NOTA
25. Suppose a rectangular field along a straight river is to be fenced. There is 300 m. of fencing available. Let the greatest area (in square meters) that can be enclosed equal
 C. What is $\frac{C}{\pi e \sqrt{2}}$, rounded to six decimal places?
 A. 931.522 B. 1863.04 C. 465.761 D. 24.9406 E. NOTA
26. In Calculus, the *derivative* of a monomial function $D(x) = ax^n$, where "a" is a constant, n is *rational*, is $D'(x) = nax^{n-1}$ (notice the tick mark after the D). The derivative of a sum is the sum of the individual derivatives. For example, the derivative of $F(x) = x^3 - 2x^2$ is $F'(x) = 3x^2 - 4x$. Let $G(x) = 3x^4 - 5x^2 + 10x - 6$. Find the derivative of G at $x = -5$.
 A. -1440 B. 7200 C. -1446 D. -1065 E. NOTA
27. Find how many natural numbers are solutions of $9x - 39 < 94$. When you write out (in letters) the number (of natural-numbered solutions), how many distinguishable permutations are there of that word?
 A. 40320 B. 24 C. 20160 D. 10080 E. NOTA
28. Find the equation of the axis of symmetry of the graph of $x = y^2 - 6y + 2$.
 A. $y = -3$ B. $y = 3$ C. $x = -7$ D. $y = -7$ E. NOTA

$$29. J(x) = 28x^{31} + 5x^{30} - 6x^{29} + 29x^{28} + x^{27} - 9x^{26} + 2x^{25} + 3x^{24} - 3x^{23} + 26x^{22} + 5x^{21} - 6x^{20} - 5x^{19} + 6x^{18} + 6x^{17} + 6x^{16} + 6x^{15} + 6x^{14} - 6x^{13} + 6x^{12} + 6x^{13} + 6x^{12} + 10x^{11} + 5x^{10} - 67x^9 - 8x^8 + 23x^7 + 100x^6 - 99x^5 - 98x^4 + 32x^3 - 2x^2 + 5x^5 + k.$$

Find the minimum number of real roots of $J(x)$.

- A. 1 B. 0 C. 31 D. 16 E. NOTA

30. What is the minimum distance between the point $(1, 8)$ and the graph of

$$9x^2 + 4y^2 - 18x + 16y - 11 = 0?$$

- A. 5 B. 6 C. 7 D. 8 E. NOTA

WORK THE FOLLOWING TIEBREAKER IN THE WHITE PORTION ON THE BACK OF THE SCANTRON SHEET.

TIE BREAKER

$$K(x) = \frac{4}{x^2} - \sqrt{x}. \quad K'(x) = L(x). \quad \text{What is the value of } L(9)? \quad \text{Note: Refer to question 26.}$$