Mu - Limits Topic Test Solutions

1. B $\lim_{x\to\pi} \frac{\tan^2 x}{1+\sec(x)} = \frac{0}{1+1} = 0$

2. C $\frac{(x+3)\cancel{(x+2)}}{\cancel{x+2}}$ $(-2,1)$

3. $\lim_{n\to\infty}\left|\frac{\frac{(n+1)!}{(n+1)^{n+1}}\cdot x^{n+1}}{\frac{n!}{n^n}x^n}\right| = \left|\frac{(n+1)!\,x^{n+1}}{(n+1)^{n+1}}\cdot\frac{n^n}{n!\,x^n}\right| = \left|\frac{(n+1)x\cdot n^n}{(n+1)^{n+1}}\right|$

D

$= \lim_{n\to\infty}\left|\frac{x n^n}{(n+1)^n}\right| = \left|\left(\frac{n}{n+1}\right)^n x\right|$

$= \lim_{n\to\infty}\left|\left(\frac{n}{n+1}\right)^n\right||x| = \frac{1}{e}|x| < 1$

$|x| < e$

$-e < x < e$

radius $= e$

note: $\lim_{p\to\infty}\left(1+\frac{1}{p}\right)^p = \lim_{p\to\infty}\left(\frac{p+1}{p}\right)^p = e$

$\therefore \lim_{p\to\infty}\left(\frac{p}{p+1}\right)^p = \frac{1}{e}$

4. $\lim_{x\to 1}\frac{\ln x - x + 1}{x^3 - 3x + 2} = \frac{0}{0}$

C

L'Hopital's Rule

$\lim_{x\to 1}\frac{\frac{1}{x}-1}{3x^2-3} = \lim_{x\to 1}\frac{-\frac{1}{x^2}}{6x} = \lim_{x\to 1}\frac{-1}{6x^3} =$

$-1/6$

5. $\lim_{x\to 8}\frac{\cancel{(x-8)}(x^2+8x+64)}{\cancel{(x-8)}} = 3(64) = 192$

B

6. $\lim_{x\to 3}\frac{x^2(x-3)-9(x-3)}{-3(x^2-6x+9)} = \frac{(x-3)(x^2-9)}{-3(x-3)(x-3)} = \frac{\cancel{(x-3)}\cancel{(x-3)}(x+3)}{-3\cancel{(x-3)}\cancel{(x-3)}} = \lim_{x\to 3}\frac{x+3}{-3}$

B

$= \frac{6}{-3} = -2$

7. Average Rate of Change $= \dfrac{\Delta y}{\Delta x} = \dfrac{3x^2-65x-400}{x+5} = \dfrac{\cancel{(x+5)}(3x-80)}{\cancel{x+5}}$

$\boxed{B}$ $\lim\limits_{x\to -5} (3x-80) = -95$

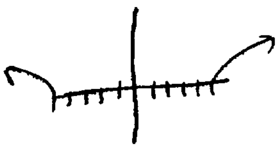
8. $f(x) = \dfrac{x+5}{(x+4)(x+1)}$ $\boxed{C}$

9. $\lim\limits_{x\to 3^-} f(x) = 15.5$ $\lim\limits_{x\to 3^+} f(x) = 15.5$ $\lim\limits_{x\to 3} f(x) = 15.5$

$\boxed{A}$ $f(3) = \frac{31}{2} = 15.5$

10. $\lim\limits_{x\to 0} \dfrac{x}{\tan x} = \dfrac{0}{0}$ L'Hopital's Rule $\lim\limits_{x\to 0} \dfrac{1}{\sec^2 x} = \dfrac{1}{1} = 1$

$\boxed{C}$

11.  $\boxed{D}$

12. vertical asymptote at $x=1$

$\boxed{A}$ $\lim\limits_{x\to 1^-} \dfrac{-2}{x-1} = \infty$

13. Look at graph: $\boxed{A}$

14. $\boxed{C}$ (15) $\lim\limits_{x\to\infty} \dfrac{2|x|}{x^2} = 0$ (E)

16. $= \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(1 + \dfrac{2i}{n} + \dfrac{i^2}{n^2}\right)\dfrac{2}{n} = \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(\dfrac{2}{n} + \dfrac{4i}{n^2} + \dfrac{2i^2}{n^3}\right)$

$\boxed{C}$ $= \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{2}{\cancel{n}}\cancel{n} + \dfrac{4}{n^2}\dfrac{(n)(n+1)}{2} + \dfrac{2}{n^3}\dfrac{(n)(n+1)(2n+1)}{6}$

$2 + 2 + \frac{4}{6} = 4\frac{2}{3}$

17. $\boxed{D}$

$|2x-1-3| < .01$

$|2x-4| < .01$

$|x-2| < \frac{.01}{2}$

$|x-2| < .005$

18. $\boxed{A}$

19. $\boxed{C}$

$2x = 5\sin\theta$

$x = 5/2 \sin\theta$

$dx = 5/2 \cos\theta \, d\theta$

$\int \sqrt{25 - 25\sin^2\theta} \cdot 5/2 \cos\theta \, d\theta$

$5/2 \int \sqrt{25\cos^2\theta} \cdot \cos\theta \, d\theta$

$5/2 \cdot 5 \int \cos^2\theta \, d\theta$

$\frac{25}{2} \left[\cos^2\theta \, d\theta \right]_{\cancel{\pi}/6}^{\pi/2}$

$2.5 = 5/2 \sin\theta$

$1 = \sin\theta$

$\pi/2 = \theta$

$1.25 = 2.5 \sin\theta$

$1/2 = \sin\theta$

$\pi/6 = \theta$

20. $\boxed{C}$

$\lim_{x\to 0} \left(\frac{1}{x}\right)^x$

$y = \left(\frac{1}{x}\right)^x$

$\lim_{x\to 0} \ln y = \lim_{x\to 0} x \ln\left(\frac{1}{x}\right)$

$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{\ln \frac{1}{x}}{\frac{1}{x}} = \frac{\infty}{\infty}$

L'Hopital's Rule

$= \lim_{x\to 0} \frac{\frac{-1/x^2}{\frac{1}{x}}}{-1/x^2} \quad \frac{-1}{x} \cdot \frac{-x^2}{1} = x = 0$

$\lim_{x\to 0} \ln y = 0$

$e^0 = y \quad \boxed{y = 1}$

21. $\lim_{x\to\infty} \frac{\ln x}{e^x} = \frac{\infty}{\infty}$

$\boxed{A}$

L'Hopital's Rule

$\lim_{x\to\infty} \frac{\frac{1}{x}}{e^x} = \frac{1}{xe^x} = 0$

22. 2, 6, 12, 20, ... ~~d~~diverges

D $\frac{n!}{(n-2)!} = \frac{n(n-1)(n-2)!}{(n-2)!} =$ $n(n-1)$ diverges

23. $\lim_{\Delta x\to 0} \frac{(x+\Delta x)^2+y^2-2(x+\Delta x) - x^2-y^2+2x}{\Delta x}$

B $\frac{\cancel{x^2}+2x\Delta x+(\Delta x)^2+\cancel{y^2}-\cancel{2x}-2\Delta x-\cancel{x^2}-\cancel{y^2}+\cancel{2x}}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{\cancel{\Delta x}(2x+\Delta x-2)}{\cancel{\Delta x}} =$ $2x+0-2 = 2x-2$

24. $\lim_{t\to 3}\left(\frac{t-3}{t^2+9} i + \frac{t+3}{t^2+9} j + \frac{1}{t+3} k\right) = \frac{0}{18} i + \frac{1}{3} j + \frac{1}{6} K$

$\frac{1}{3}j + \frac{1}{6}K$

$\boxed{D}$

25. B

26. $\int_0^3 \frac{dx}{(x-1)^{2/3}} = \lim_{b\to 1^-}\int_0^b \frac{dx}{(x-1)^{2/3}} + \lim_{b\to 1^+}\int_b^3 \frac{dx}{(x-1)^{2/3}}$

D

$\lim_{b\to 1^-}\left[3\sqrt[3]{x-1}\right]_0^b + \lim_{b\to 1^+}\left[3\sqrt[3]{x-1}\right]_b^3$

$(0 - {-3}) + 3\sqrt[3]{2} - 0 =$ $3+3\sqrt[3]{2}$

27. $\lim_{x \to 3} f(x) = \frac{(x+5)\cancel{(x-3)}}{\cancel{x-3}} = 8$

$\boxed{D}$

$f(3) = k^3 - 1 = 8$

$k^3 = 9$

$k = \sqrt[3]{9}$

28. $\boxed{A}$ Look at graph

29. $\lim_{x \to 0} \frac{\tan 3x}{\sin 5x} = \frac{\sin 3x}{\cos 3x} \cdot \frac{1}{\sin 5x}$

$\boxed{B}$

$= \lim_{x \to 0} \frac{\sin 3x}{\sin 5x} \cdot \frac{1}{\cos 3x} = \frac{3}{5} \cdot 1 = 3/5$

$\lim_{x \to 0} \frac{3}{5} \cdot \frac{\sin 3x}{3x} \cdot \frac{5x}{\sin 5x} = \lim_{x \to 0} 3/5(1)(1) = 3/5$

30. $.2 < y_0 < .6$

$\boxed{E}$

$\cos x = .2$ $\cos x = .6$

$x = 1.37$ $x = .927$

But $x \in [-\pi/2, 0]$

Quad. IV

$(-.927, -1.37)$