Mu Division
Probability Topic Test
Solution key

1. $\boxed{A}$

2. $\boxed{D}$ Area must be one since, according to the theory of probability, the sum of all possible probabilities of an event must be 1. $\boxed{D}$

3. $\boxed{C}$ $y = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$

$y' = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \cdot (-x) = 0$

y': $+$ $-$ at 0

max ht when $x = 0$, $y = \frac{1}{\sqrt{2\pi}} e^0 = \frac{1}{\sqrt{2\pi}} \doteq .4$

4. $\boxed{C}$ $\int_0^1 (x^3+1)\,dx = \left[\frac{x^4}{4} + x\right]_0^1 = 5/4$ $\quad \frac{1}{5/4} = 4/5 = .8$

5. $\boxed{A}$ $\int_0^x (t^3 - 2t + 3)\,dt = \left[\frac{t^4}{4} - t^2 + 3t\right]_0^x = \frac{x^4}{4} - x^2 + 3x - 0$

$\frac{x^4}{4} - x^2 + 3x = 24\left(\frac{1}{6}\right) = 4$

$x^4 - 4x^2 + 12x - 16 = 0$

use calculator

$x \doteq 1.63$

6. $\boxed{A}$ $\int_0^\pi \sin x\,dx = \left[-\cos x\right]_0^\pi = 1 - {}^-1 = 2$

$\int_{\pi/6}^{5\pi/6} \sin x\,dx = \left[-\cos x\right]_{\pi/6}^{5\pi/6} = \frac{\sqrt{3}}{2} + {}^+\frac{\sqrt{3}}{2} = \sqrt{3}$

ratio $= \frac{\sqrt{3}}{2}$

7. $\boxed{E}$ $P = \frac{4}{36} = 1/9$

$3x^2 + 2y = 7$

$6x + 2y' = 0$

$2y' = -6x$

$y' = \frac{-6x}{2} = -3x$ at $x = P$

$y' = -3\left(\frac{1}{9}\right) = \frac{-1}{3}$

8.) $\boxed{B}$

9.) $\boxed{B}$

P(brown on 1st draw) = $\frac{1}{3}$ P(red on 1st) = $2/3$

P(brown on 2nd draw) = $2/4 = 1/2$ P(red on 2nd) = $1/2$

P(brown on 3rd draw) = $3/5$ P(red on 3rd) = $2/5$

P(brown on 4th draw) = $4/6 = 2/3$ P(red on 4th) = $1/3$

P(3 B and 1 R) =

BBBR + BBRB + BRBB + RBBB

$\frac{1}{3}(\frac{1}{2})(\frac{3}{5})\frac{1}{3} + (\frac{1}{3})(\frac{1}{2})(\frac{2}{5})(\frac{2}{3}) + \frac{1}{3}(\frac{1}{2})(\frac{3}{5})(\frac{2}{3}) + \frac{2}{3}(\frac{1}{2})(\frac{3}{5})(\frac{2}{3})$

$\frac{1}{30} + \frac{2}{45} + \frac{1}{15} + \frac{2}{15} = \frac{3}{90} + \frac{4}{90} + \frac{6}{90} + \frac{12}{90} = \frac{25}{90}$

$= \frac{5}{18}$

10. $\boxed{C}$

P(2 tails) = $1/4$

P(1 tail) = $2 \cdot 1/4 = 1/2$

P(0 tails) = $1/4$

$1/4(10) + 1/2(5) + 1/4(1) = \frac{10}{4} + \frac{5}{2} + \frac{1}{4} = \frac{11}{4} + \frac{10}{4} = \frac{21}{4}$

\$5.25

11. $\boxed{C}$

4 · (GGGR)

$4 \cdot \left[\frac{7}{12}\right]\left[\frac{6}{11}\right]\left[\frac{5}{10}\right]\left[\frac{5}{9}\right] = 35/99$

12 $\boxed{C}$

1 − P(none of the 5 received an A)

$1 - \frac{{}_{35}C_5}{{}_{45}C_5} = 1 - \frac{324{,}632}{1{,}221{,}759} = .7343$

13. $\boxed{A}$

$1 - [\cancel{P(0)} + P(1) + P(2) + P(3) + P(4)]$

$1 - \left[\frac{7}{15} + \frac{8}{15}\cdot\frac{7}{14} + \frac{8}{15}\cdot\frac{7}{14}\cdot\frac{7}{13} + \frac{8}{15}\cdot\frac{7}{14}\cdot\frac{6}{13}\cdot\frac{7}{12}\right]$

$1 - \left[7/15 + \frac{4}{15} + \frac{28}{195} + \frac{14}{195}\right] =$

$.05128 = 2/39$

14. $\boxed{C}$ $\dfrac{2\cdot 7\,[1\cdot 1\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1]}{8!} = \dfrac{2\cdot 7!}{8!} = \dfrac{2}{8} = \dfrac{1}{4}$

15. $\boxed{B}$ $P(\text{flaw}) = \frac{8}{20} = \frac{2}{5}$ $P(\text{good}) = \frac{3}{5}$

$$\frac{{}_8C_2 \cdot {}_{12}C_3}{{}_{20}C_5} = \frac{28 \cdot 220}{15504} = .3973$$

$$= \frac{385}{969}$$

16. $\boxed{A}$ $\dfrac{{}_5C_5 \cdot {}_{10}C_5}{{}_{15}C_{10}} = \dfrac{1(252)}{3003} = \dfrac{12}{143}$

17. $\boxed{D}$ P(not SR and No 2nd lang)

	Sr	Jr	Soph	Freshmen
no 2nd lang	14	25−x 21	30	17
take 2nd lang	6	x 4	5	3

$(14+25-x) = \frac{7}{9}(45)$

$39 - x = 35$

$4 = x$

P(not SR and No 2nd lang) = P(Not SR) · P(No 2nd lang / Not SR)

$= \frac{80}{100} \cdot \frac{68}{80} = .68$

18. $\boxed{B}$ 3rd try = toss a coin

P(T and T) + P(H and 4or5)

$\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$

19. $\boxed{A}$ P(a, e, i, o, u)

P(vowel in 1st box) = 3/10

P(vowel in 2nd box = 2/16

$\frac{1}{2}(3/10) + \frac{1}{2}(2/16) = \frac{3}{20} + \frac{1}{16} = \frac{17}{80}$

20. $\boxed{A}$ $P(W_T | W) = \dfrac{P(W_T \text{ and } W)}{P(W)} =$

If white is transferred from A to B, urn B now has 4W, 2B
If white is not transferred (black is), urn B now has 3W, 3B

$P(W_T \text{ and } W) = \frac{2}{4}\left(\frac{4}{6}\right) = \frac{1}{2} \cdot \frac{2}{3} = 1/3$

$P(W) = 1/3 + 2/4(3/6) = 1/3 + 1/4 = 7/12$

$\dfrac{\frac{1}{3}}{7/12} = \frac{1}{3} \cdot \frac{12}{7} = \frac{4}{7}$

21. $\boxed{B}$ $P(B | \text{def}) = \dfrac{P(B \text{ and def})}{P(\text{def})} = \dfrac{1/4 \cdot 3/100}{1/4(5/100) + 1/4\left(\frac{3}{100}\right) + \frac{1}{4}\left(\frac{4}{100}\right) + \frac{1}{4}\left(\frac{8}{100}\right)}$

$= \dfrac{\frac{3}{400}}{\frac{1}{4}\left(\frac{1}{5}\right)} = \frac{3}{400} \cdot 20 = \frac{3}{20} = .15$

22. $\boxed{B}$ $P(X \geq 11 \text{ correct answers})$

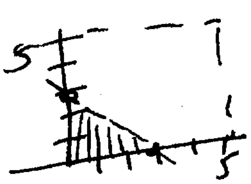
$P(X \geq 11) = 1 - P(X \leq 10) = 1 - .9961 = .0039 \doteq .004$ $\boxed{B}$

use TI-83 Calculator " binomcdf)

23. $\boxed{C}$ $P(2\ 6\text{'s out of } 17 \text{ rolls}) \cdot P(6 \text{ on } 18^{th} \text{ roll})$

${}_{17}C_2\left(\frac{1}{6}\right)^2 (5/6)^{15} \cdot \frac{1}{6} = 136\left(\frac{1}{36}\right)\left(\frac{5}{6}\right)^{15}\left(\frac{1}{6}\right) = .0409 = .041$

24. $\boxed{D}$



$\dfrac{\text{area A}}{\text{area sq}} = \dfrac{1/2(9)}{25} = \dfrac{4.5}{25} = .18$

25. $\boxed{D}$

A B C or A B C A B or A B C A B C A B

T H or T T T T H or T T T T T T T H

$\frac{1}{2}\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^7 \cancel{\tfrac{1}{2}} + \dots$

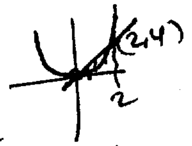
infinite geom series: $r = \left(\frac{1}{2}\right)^3$

$$S = \frac{a_1}{1-r} = \frac{1/4}{1-1/8} = \frac{\frac{1}{4}}{7/8} = \frac{1}{4}\cdot\frac{8}{7} = \frac{2}{7}$$

26. Every 4 years – there is one Feb 29.

$\boxed{B}$ $P($~~Feb~~ 29 B'day$) = \frac{1}{365(3)+366} = \frac{1}{1461}$

$$P(\text{at least one}) = 1 - P(\text{none})$$
$$= 1 - \left(\frac{1460}{1461}\right)^n > .5$$
$$.5 > \left(\frac{1460}{1461}\right)^n$$
$$\ln .5 > n \ln\left(\frac{1460}{1461}\right)$$
$$-.693147 > n(-.000684697)$$
$$1012.34 < n$$
$$1013$$

Tie Breaker: 

$\boxed{\frac{256}{21}\pi}$

$$V = 2\pi\int_0^2 (2-x)(2x-x^2)\,dx = 2\pi\left[2x^2 - \frac{4x^3}{3} + \frac{x^4}{4}\right]_0^2$$
$$2\pi\int_0^2 (4x - 4x^2 + x^3)\,dx = 2\pi\left(8 - \frac{32}{3} + 4 - 0\right] = 2\pi\left(\frac{4}{3}\right)$$

Volume $\frac{8\pi}{3}$

$${}_8C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^3 = 56\left(\frac{1}{256}\right) = \frac{7}{32}$$

$$\text{Ratio} = \frac{\frac{8\pi}{3}}{7/32} = \frac{8\pi}{3}\cdot\frac{32}{7} = \boxed{\frac{256}{21}\pi}$$