C 1. I. $\sqrt{2}+i\sqrt{11}$ nope , II. $-5(-i)=5i$ yep, III. $-711^{82}i-1110^{82}$ nope, IV. $i\sqrt{x^2-4}$ yep
II and IV only

B 2. $a^3=27; a=3; -b^3=-64, b=4; a+b=7$

C 3. $25+\frac{i}{2}+\frac{1}{3}-\frac{i}{2}=\frac{76}{3}$

C 4. $30i-6\sqrt{2}+5\sqrt{2}+2i=32i-\sqrt{2}$

A 5. $\frac{4+i}{2-3i}\bullet\frac{2+3i}{2-3i}=\frac{8+12i+2i-3}{13}=\frac{5}{13}+\frac{14}{13}i$

D 6. $2x-ix+5yi+y=(2x+y)=i(-x+5y)=7+13i;\begin{cases}2x+y=7\\-x+5y=13\end{cases}$

$x=2, y=3; y!+\sin\frac{\pi}{x}=6+1=7$

C 7. Note that f(1)=0. Thus $f(x)=(x-1)(5x^2+2x+2)$. Since $4-4(5)(2)<0, 5x^2+2x+2$ has no real solutions. Then sum is $\frac{-b}{1}=-\frac{2}{5}$.

B 8. $r^2-13r+1=0; r^2+1=13r, r+\frac{1}{r}=\frac{r^2+1}{r}=\frac{13r}{r}=13$

B 9. Region is a circle centered at $2+3i$ with radius 4. Area is 16π .

A 10. $\left(2e^{\frac{9\pi}{8}i}\right)^4=16e^{\frac{9\pi}{2}i}=16i$

D 11. Let $z=a+bi$,where $b\neq 0, |z|=1$. $\frac{z-1}{z+1}=\frac{(a+bi-1)(a-bi+1)}{(a+bi+1)(a-bi+1)}=\frac{a^2+b^2-1+2bi}{(a+1)^2+b^2},(a^2+b^2=1),$

$\frac{2bi}{(a+1)^2+b^2}$, clearly purely imaginary

A 12. Note that a Hermitian matrix must have real entries on the main diagonal otherwise $z\neq\bar{z}$. That leaves choice A to be tested. We get $\begin{pmatrix}3 & 2-i & 1-2i\\ 2+i & -1 & -3i\\ 1+2i & 3i & 4\end{pmatrix}=\begin{pmatrix}3 & 2+i & 1-2i\\ 2-i & -1 & -3i\\ 1+2I & 3I & 4\end{pmatrix}=a$

so it's Hermitian.

E 13. $(-2-2i\sqrt{3})e^{i\theta}=4e^{\frac{1802\pi}{3}i}=4e^{\frac{2\pi}{3}i}, 2\pi$ coterminal to $\frac{1802\pi}{3}i$, $\not{4}e^{\frac{4\pi}{2}i}e^{i\theta}=\not{4}e^{\frac{2\pi}{3}i}\rightarrow e^{i\theta}=^{-\frac{2\pi}{3}i}$

$\rightarrow\theta--\frac{2\pi}{33}+2\pi r$. Smaller θ in abs. value is $-\frac{2\pi}{3}$.

D 14. By symmetry, there are the same number of roots in quadrant I. $0<\frac{360}{57}n<90\rightarrow 0<n<14.25$.

There are 14 integers inside this interval.

B 15. Area $=\left|\frac{1}{2}\begin{pmatrix}2 & 3 & 1\\ 4 & -5 & 1\\ 1 & 1 & 1\end{pmatrix}\right|=6$

D 16.

$x = e^{\frac{73\pi}{180}i} = \cos 73 + i\sin 73, y = e^{\frac{17\pi i}{180}} = \cos 17 + i\sin 17,$
$\text{Im}(x)\text{Re}(y) + \text{Re}(x)\text{Im}(y) = \sin 73\cos 17 + \cos 73\sin 17,\ \sin(73+17) = \sin 90 = 1$

C 17. $3^4 \equiv 1 \bmod 4$ so $3^{909} = 3^{908} \bullet 3$, $\left(3^4\right)^{227}(3) = 1(3) \bmod 4 \equiv 3 \bmod 4$. Thus, $i^n = i^3 = -i$.

B 18. Notice that $f^{(even)}(z) = z$ and $f^{(odd)}(z) = \bar{z}$. Listing the sum out and letting $z = 1+i$, we get $(f^{(0)}(z) - f^{(1)}(z)) + (f^{(3)}(z) - f^{(4)}(z)) + (f^{(6)}(z) - f^{(7)}(z)) + \ldots$ Changing to conjugate notation we have $(z - \bar{z}) + (\bar{z} - z) + (z - \bar{z}) + \ldots$ Notice that every group of two terms cancel each other out in a telescoping like manner. Since n ranges from 0 to 2003, we have 2004 terms (or 1002 groupings) and thus everything cancels out, making the sum 0.

C 19. $-i$ is also a root. So $-i, i, 2,$ and w are the roots. Sum of roots = 2+w=$-\frac{a}{2}$, product of roots $= 2\text{w} = \frac{3}{2}$, so w = $\frac{3}{4}$ and a=$-\frac{11}{2}$.

A 20. Since $z^3 = 1 \to (z-1)(z^2+z+1) = 0$. If $z \neq 1$, then $)z^2 + z + 1) = 0.$ So $(1 - z - z^2)(1 + z - z^2) = ((-z - z^2) - z + z^2)((-z^2) - z^2)(-2z)(-2z^2) = 4z^3 = 4$.

B 21. I is obviously false, as any real number is also a complex number. II is not true because $|\sin(5i)| = i\sinh 5 \approx 74.20 > 1$. III is not correct because the range of Arctan is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, and therefore will not produce the correct angle for complex numbers in, say, the third quadrant. The expression in IV reduces to $i^a = i^b$, but a and b aren't necessarily equal (take (a,b) = (4,8), for example). None of the statements are always true.

A 22. $1 + \cos 4 = 2\cos^2 2$ and $\sin 4 = 2\sin 2\cos 2$, so $(\cos 4 + i\sin 4 + 1)^{2003} = (2\cos^2 2 + 2i\sin 2\cos 2)^{2003} =$ $(2\cos 2(\cos 2 + i\sin 2))^{2003} = (2^{2003}\cos^{2003} 2)(\cos 4006 + i\sin 4006) = \text{Im}(z) = 2^{2003}\cos^{2003} 2\sin 4006$.

A 23. $f(t) = \frac{(t+2i)^2 + (t-2i)^2}{t^2+4} = \frac{2t^2 - 8}{t^2+4} = 2 - \frac{16}{t^2+4}$, where the denominator is positive so max is achieved and this is 0. $\max|f(t) = 2|$. Note, this is achieved when t=0.

C 24. $\frac{r}{r-1} + \frac{s}{s-1} + \frac{t}{t+1} = 3 + \frac{1}{r-1} + \frac{1}{s-1} + \frac{1}{t-1}$ - sum of the reciprocals of the roots of a polynomial whose roots are $r-1, s-1, t-1$r. This is precisely $f(x+1)$. $f(x+1) = x^3 + 5x^2 - 2$. Sum of reciprocals = $\frac{5}{2}$, $3 + \frac{5}{2} = \frac{11}{2}$.

B 25. $\frac{1}{1+i} = \frac{1-i}{2} . i^{\left(\frac{1-i}{2}\right)} = \left(e^{\frac{\pi}{2}i}\right)^{\frac{1-i}{2}} = e^{\frac{\pi}{4}(1+i)} = e^{\frac{\pi}{4}} e^{\frac{\pi}{4}i} = e^{\frac{\pi}{4}}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$

A 26. $\sin(a+bi) = \sin a \cos(bi) + \cos a \sin(bi)$, $\sin a \cosh b + i \cos a \sinh b = \sin a\left(\frac{e^b + e^{-b}}{2}\right) + i\cos\left(\frac{e^b - e^{-b}}{2}\right)$,

let a=3 and b=4 to get A

D 27. $x + \frac{1}{x} = \sqrt{3} \rightarrow x^2 - x\sqrt{3} + 1 = 0 \rightarrow x = e^{\frac{\pi}{6}i}$,

$x^n + x^{-n} = \left(e^{\frac{\pi}{6}i}\right)^n + \left(e^{-\frac{\pi}{6}i}\right)^n = \cos\frac{\pi n}{6} + i\sin\frac{\pi n}{6} + \cos\frac{\pi n}{6} - i\sin\frac{\pi n}{6}$,

$2\cos\frac{\pi n}{6} \rightarrow n = 6000 \rightarrow 2\cos 100\pi$. Be careful, the greatest integer less than this is 1.

C 28. Let $f(x) = x^3 + Bx + C$ be the polynomial with roots of a, b, and c. Let $S_n = a^n + b^n + c^n$.

Using Newton sums, we find that $S_3 = -3C, S_4 = 2B^2, S_7 = -7B^2C$. Hence $\frac{S_3 S_4}{6} = \frac{S_7}{7}$,

thus $S_3 S_4 = \frac{6}{7} S_7 = 6/$

A 29. If $0 < t < \frac{\pi}{2}$ then tan t >0. By the quadratic formula, $x = \frac{-\tan t \pm \sqrt{\tan^2 t - 4(\tan^2 t)}}{2\tan^2 t} =$

$\left(\frac{-1 \pm i\sqrt{3}}{2}\right)\cot t.\ r = \frac{-1 \pm i\sqrt{3}}{2}\cot t = e^{\frac{2\pi}{3}i}\cot t, s = e^{-\frac{2\pi}{3}i}\cot t\ .\ r^n + s^n = e^{\frac{2\pi n}{3}i}\cot^n t + e^{-\frac{2\pi n}{3}i}\cot^n t =$

$2\cos\frac{2\pi n}{3}\cot^n t$. If n=6000, then $r^{6000} + s^{6000} = 2\cos 4000\pi \cot^{6000} t = 2\cos^{6000e}$

A 30. Start at origin of complex plane, where left is upwards and right is downwards. On the first state,

the tourist is on the complex number $x = 100 + 100e^{-\frac{\pi}{3}i} + 100e^{-\frac{2\pi}{3}i} = 100(1 - i\sqrt{3})$. Notice how we're treating complex numbers like vectors. Notice that the path transversal on a particular stage is just the previous one rotated by $\frac{\pi}{3}$. Thus, after 2003 stages, the distance from (0,0) is

$$\left| x + xe^{\frac{\pi}{3}i} + xe^{\frac{2\pi}{3}i} + \ldots + xe^{\frac{2002\pi}{3}i} \right| = |x| \left| \frac{e^{\frac{2003\pi}{3}i} - 1}{e^{\frac{\pi}{3}i} - 1} \right| = 200$$