NUMBER THEORY
ALPHA ALTANTA, 2003

UNITS' DIGIT

C 1. $3^1=3$; $3^2=9$, $3^3 \Rightarrow 7$, $3^4 \Rightarrow 1$

REPEATS IN 4's.

5 is not included.

E 2. (a, b) where $a<b$ and $a \cdot b = 30$

$1 \cdot 30$	$-30 \cdot -1$
$2 \cdot 15$	$-15 \cdot -2$
$3 \cdot 10$	$-10 \cdot -3$
$5 \cdot 6$	$-6 \cdot -5$

$\boxed{8}$

D 3. $a+k=a \Rightarrow k=0$

B 4. $a+b=12$

largest integer for b if a and b are integers $\boxed{11}$

A 5.
$$\begin{array}{r} A2C8 \\ -\ A79 \\ \hline C66B \end{array} \Rightarrow \begin{array}{r} A2C8 \\ -\ A79 \\ \hline C66\ B=9 \end{array}$$

4

$$\begin{array}{r} A2C8 \\ -\ A79 \\ \hline C669 \end{array} \Rightarrow$$

since I borrowed

$c-1-7=6$

$c-8=6$

$c=14$

$$\begin{array}{r} A248 \\ -\ A79 \\ \hline 4669 \end{array} \Rightarrow$$

$6+A=1$

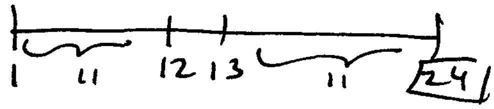
or since I borrowed.

$6+A=11$

$A=5$

$$\begin{array}{r} 5248 \\ -\ 579 \\ \hline 4669 \end{array}$$

$A+B+C = 5+9+4 = \boxed{18}$

C. 6. 

E. 7. I. $E+E = E$ TRUE

II. $2+3=5$ yes
$3+5=8$ no FALSE

III. "+" + "+" = "+" TRUE

$\sqrt{1+2+3+x}$ = integral value

B. 8. if $x=3$ then $\sqrt{6+3} = \sqrt{9} =$

if $x=-2$ then $\sqrt{6+(-2)} = \sqrt{4} = 2$

if $x=-5$ then $\sqrt{6+(-5)} = \sqrt{1} = 1$

if $x=-6$ then $\sqrt{6+(-6)} = \sqrt{0} =$

D 9. $x>0$, $y>0$ x is even
y is odd.

When must be odd?

xy	$x+2y$	x^4	y^x
$E \cdot O$	$E+E$	E^{o}	O^{E}
E	E	E	O

A 10. $\gcd(24x^4y, 30x^3y^5)$

$gcf = ax^by^c$

$= 6x^3y$

$a+b+c = 6+3+1 = \boxed{10}$

C 11. $(387, a42) \div 3$

$3+8+7+a+4+2$

$24+a$

If mult. of 3 then $a=0$, $a=3$, $a=6$, or $a=9$. 4 solutions

THEORY - ALPHA
ALTANTA 2003

21. B

$2n^2 = \frac{1}{5}n^3$

$10n^2 = n^3$

$10n^2 - n^3 = 0$

$n^2(10-n) = 0$

$n = 10$

$2 \cdot 10^2 = 200$

$\frac{1}{5} \cdot 10^3 = 200$

22. D

$a * b$ = sum of twins primes between 25 and 75.

29, 31
41, 43
59, 61
71, 73

SUM = $\boxed{408}$

23. A

Smallest in base$_7$ = $100_7 = 49_{10}$

largest in base$_4$ = $333_4 = 63_{10}$

$63_{10} - 49_{10} = 14_{10}$

$14_{10} + 1_{10} = 15_{10}$

$\boxed{15}$

24. C. DIVISIBILITY BY 99.

11 and 9.

If $(A+B+80) \Rightarrow$ divisible by 9 the $A+B=1$ or $A+B=10$

$(B+4+9+2+8+8+5+4+3) - (4+6+7+1+9+4+6+A) =$ mult. of 11.

$(B+43)-(37+A) = 0$ or ± 11

$B - A + 6 = 0$

$B - A = -6$ or $A+B=10$

$B - A + 6 = 11$

$B - A = 5$

If $A+B=10$, $-A+B=5$: $2B=15$; or $A+B=10$, $-A+B=-6$: $2B=4$

24. cont. C

If $B=2$, then $A+B=10$, then $A=8$

$(A, B) = (8, 2)$

$8+2=10$

25. E

$100! = 24$ zeros.

REPEATING PATTERN

\# of fives

5-1
10-1
15-1
20-1
25-2
6

30-50: 6
55-75: 6
80-100: 6

105! has 24+1 zeros or 25

105, 106, 107, 108, 109 share the same property. $\boxed{5}$ E

26.

$\boxed{1}$ $\boxed{4}$ $\boxed{9}$ $\boxed{16}$ $\boxed{25}$ $\boxed{36}$

1·1 | 2·2, 4·1, 1·4 | 3·3, 9·1, 1·9 | 4·4, 2·8, 8·2 | 5·5 | 6·6, 4·9, 9·4

$\boxed{49}$ 7·7 | $\boxed{64}$ 8·8 | $\boxed{81}$ 9·9

$\boxed{D}$

There are 17 unique pairs.

27. $a > 20$, $b > 20$, $c > 20$

One has an odd # of divisors, thus one is a perfect square. The other two are the squares of prime #s.

$5^2 + 12^2 = 13^2$

$25 + 144 = 169$

$\boxed{A}$

THEORY
ATLANTA, 2003

28. $x > y > z > 0$

~~ARITHMETIC~~ sequence.

$x + d = y \quad y + d = z$

$d = y - x \quad d = z - y$

$y - x = z - y$

$2y = z - x$

$y = \frac{z - x}{2}$

The difference had to be even.
So I worked odd squares.

(C)

~~49 + 169 =~~

49, 169, 289

$d = 120$

29. 12 divisors

(D)

$2 \cdot 6$	$3 \cdot 4$	$2 \cdot 2 \cdot 3$.
$(1+1)(1+5)$	$(1+2)(1+3)$	$(1+1)(1+1)(1+2)$
$2^5 \cdot 3$	$2^3 \cdot 3^2$	$2^2 \cdot 3 \cdot 5 = 60$
96	72	$2^2 \cdot 3 \cdot 7 = 84$
$2^5 \cdot 5$	$3^3 \cdot 2^2$	$2^2 \cdot 5 \cdot 7 = 140$
~~160~~	~~108~~	

Given: 60

Next 3 smallest: $72 + 84 + 96$

252

30. $n = ka$

d. if $k > 1$ $n > 0$ and $a > 0$.

Proper divisors - perfect squares are excluded.

$2 \cdot 3$	$3 \cdot 5$	$5 \cdot 7$
$2 \cdot 5$	$3 \cdot 7$	
$2 \cdot 7$	$3 \cdot 11$	
$2 \cdot 11$	$3 \cdot 13$	
$2 \cdot 13$	$3 \cdot 17$	
$2 \cdot 17$		
$2 \cdot 19$		
$2 \cdot 23$		

Prime factors.

$8 + 5 + 1$

$\boxed{14}$

THEORY P. 2

ATLANTA, 2003

12
A

Possible pairs

$ab = 12$ where b is odd.

$1 \cdot 1200$
$3 \cdot 400$
$5 \cdot 240$
$15 \cdot 80$
$25 \cdot 48$
$75 \cdot 16$

$\underline{\underline{75}}+16 = \boxed{91}$

largest $\underline{\underline{b}}$.

$1200 = 2^4 \cdot 3 \cdot 5^2 = 16 \cdot 75$

13. $X < 200$ X has 3 factors

B

SQUARE OF A PRIME # only.

$2 \to 4$, $3 \to 9$, $5 \to 25$, $7 \to 49$, $11 \to 121$, $13 \to 169$

4
9
25
49
121
169
377

DEL. 9 $\Rightarrow$ 9 is a mult. of 3

$377 - 9 = \boxed{368}$

14 1800

B

$1 \cdot 1800$, $2 \cdot 900$, $3 \cdot 600$, $4 \cdot 450$, $5 \cdot 360$, $6 \cdot 300$, $8 \cdot 225$, $9 \cdot 200$, $10 \cdot 180$, $12 \cdot 150$, $15 \cdot 120$, $18 \cdot 100$, $20 \cdot 90$, $24 \cdot 75$, $25 \cdot 72$, $30 \cdot 60$, $36 \cdot 50$, $40 \cdot 45$

$\boxed{\text{18 TOTAL}}$

15.
C

$(20)(25)(30)(35)(40)(45)(50)$

$2^2 \cdot 5 \quad 5^2 \quad 2 \cdot 3 \cdot 5 \quad 5 \cdot 7 \quad 2^3 \cdot 5 \quad 3^2 \cdot 5 \quad 2 \cdot 5^2$

$2^7 \cdot 5^9 \Rightarrow (2 \cdot 5)^7 \cdot 5^2 \Rightarrow 10^7 \cdot 25$

$\boxed{\text{7 zeros}}$

16.
B

5 mod 7 $\Rightarrow$ $5+12+19+26$

$\boxed{62}$

17. $\frac{4000}{n} \Rightarrow$ quotient + rem. 4

$4000 - 4 = 3996$.

$3996 \Rightarrow 2^2 \cdot 3^3 \cdot 37$

# of factors $(2+1)(3+1)(1+1)$

$3(4)(2)$

24

However 1, 2, 3 and 4 must be remo[ved]

E. $24 - 4 = \boxed{20}$

18. There are only 2 perfect squares

A between 50 + 99, namely 64 and 81. $|81-64| = |17| = \boxed{17}$

19. $LCM(a^4b^3c^2d, a^7b^5c^3d, a^5b^4c^3d^2)$ is $a^7b^5c^3d^2$

# of factors. $(7+1)(5+1)(3+1)(2+1)$

$\boxed{C}$

$8(6)(4)(3)$

$\boxed{576}$

20. The first 100 countings nos factorial has 24 zeros.

$105! = 25$ zeros
$110! = 26$ zeros
$115! = 27$ zeros

Using only factors of 2 and 5.

115, 116, 117, 118 and 119 also 27 zeros.

multiple of 3 $\Rightarrow \boxed{117}$