

Polygons/Circles – Theta Solutions National Mu Alpha Theta Convention 2003

1. $C_5 C_5 + C_5 C_4 + C_5 C_3 = 16$

2. D Sum of arcs must equal 360, making $x = 25$, $m\widehat{ABC} = 325$

3. D Triangle MCS is right with hypotenuse 10, and one leg = 8, $CM=6$, $BM = 10+6=16$

4. C ACEG is a square so to find a side of ACEB, extend segments BC, DE, FG, and AH to form a square. Use the triangle that has \overline{CE} as a hypotenuse. Let the right angle be point M. Since $CD=1$, $CM=\frac{\sqrt{2}}{2}$ and $ME = \frac{\sqrt{2}}{2} + 1$. Use Pythag to find $(CE)^2$. This is the area $2 + \sqrt{2}$.

5. A $2x \cdot 3x = 3(x+3)$. Solving makes $x=1.5$. Lengths of chords are 7.5 and 7.5. Product is 56.25.

6. B Let $m\widehat{AC} = a, m\widehat{BD} = b, m\angle BKD = \frac{b-a}{2}$,
 $m\angle BMD = \frac{a+b}{2}, \left(\frac{a+b}{2}\right) = 2\left(\frac{b-a}{2}\right)$, makes
 $3a = b$, substituting makes $\frac{2a}{3a} = \frac{a}{3a} = \frac{1}{3}$.

7. C Let a side of the square = 8 with points of tangency of the circle, isosceles right triangles are formed with legs 4 and hypotenuse $4\sqrt{2}$ - also the side of the inscribed square. Original square has area 64, remaining square has area 32. $\frac{1}{2}$ of the original was cut off.

8. E
 $m\angle OBA = m\angle OAB = 50, m\angle OBC = 130,$
 $180 - (130 + 30) = 20$

9. C Divide all terms by 4 and completing the square gives $(x+3)^2 + (y-2)^2 = 19$, area = 19π

10. B Let AB be the diameter of circle O. $OB=4,$
 $O'B=2$, area of circle O' = 4π

11. D Let arc AD = x and arc CB = y then
 $\frac{x+y}{2} = 95$ and $\frac{x-y}{2} = 25$. Solving the system gives $x=120$ and $y=70$. Let arc AC=a and arc BD=b. $a+b=360-190=170$. $a+70=2x$ and $b+70=3x$ making $a+b+140=5x$, since $a+b=170$, $310=5x$ making $x=62$. Arc BD=186 so arc BCD = $360-186 = 174$.

12. B The measure of one exterior angle is 3 degrees, divide 360 by 3 to get the number of sides which is 120. Diagonals =
 $\frac{n(n-3)}{2} = \frac{120 \cdot 117}{2} = 7020$.

13. A $AB=AC=20$. Let M be the point of tangency of segment DE. $BD=DM$ so $AD+DM=20$. $CE=EM$ so $AE+EM=20$, making the perimeter 40.

14. B Central angle = 60 since triangle is equilateral. area of sector – area of triangle = π .
 $\left(\frac{1}{6}\pi r^2 - \frac{r^2\sqrt{3}}{4} = \pi\right)$ solving gives
 $r^2 = \frac{12\pi}{2\pi - 3\sqrt{3}}, r = 5.89$

15. D $6 \cdot \left(\frac{s^2}{\sqrt{3}} 4\right) = \sqrt{108}$, solving gives $s=2$ so the diameter is 4.

16. D Using the 2 chord power theorem,
 $(2x+1)(5x+2) = (3x-1)(5x-2)$, solving gives $x=4$. Triangles are similar by AA, $AB=10, BK=11,$
 $CD=22, \frac{10}{x} = \frac{11}{22}, x = 20$.

17. B Distance from center is perpendicular bisector of chord. Draw right triangle. Legs are 5 and 5 so hypotenuse = $5\sqrt{2}$, area = 50π .

18. B $119 = \frac{n(n-3)}{2}, 238 = n^2 - 3n$, solving gives $n=17$.

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19. C In triangle ABP, draw the altitude from P to segment AB, label the foot of the altitude M. Using one of the right triangles formed and Pythagorean theorem, $PM = \sqrt{x^2 - 36}$. $PM + (P \text{ to segment DC})$ equals a side of the square. $\sqrt{x^2 - 36} + x = 12$. Solving $x=7.5$.

20. B Let the side of the square be s . radius of one circle is $\frac{1}{4}s$. Area of the four circles is

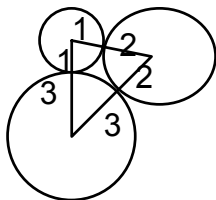
$$4 \cdot \left(\frac{1}{4}s\right)^2 \pi = \frac{1}{4}s^2\pi . \text{ Area of square is } s^2, \text{ so ratio is } \frac{\pi}{4}.$$

21. E Measure of central $\angle POQ = 108$. Area of sector is $\frac{108}{360} \cdot 100\pi = 30\pi$.

$$22. B \frac{2}{5} \cdot 20\pi = 8\pi$$

23. D Side of triangle is 12 so area of the triangle is $36\sqrt{3}$ so $36\sqrt{3} = 6 \cdot \frac{s^2\sqrt{3}}{4}$ makes $s=2\sqrt{6}$.

24. A Sides of triangle are 3, 4, 5, so it's a right triangle and area is 6.



25. C Interior angle of a regular pentagon is 108, regular decagon is 144, ratio is 3:4

26. D $m\angle EFA = 120, m\angle FAE = m\angle FEA = 30,$
 $m\angle AFG = 30$ since $AG=AF, m\angle AGF = 120,$
 $m\angle AGB = 60$.

27. D Diagonals are perpendicular and bisect each other making 6,8,10 right triangles. $P=40$.

28. B $2x+3x+4x+6x=360, x=24$. Smallest interior angle is 48, largest exterior angle is 132.

29. C Angle of hexagon = 120, angle of square = 90. Since the sum of the angles at one vertex must be 360: $360-(120+90)=150$. Dodecagon.

30. C Diagonals are perpendicular and bisect forming 30-60-90 triangles. Ratio of diagonals is $x\sqrt{3} : x = \sqrt{3} : 1$.