Mu Alpha Theta Convention 2003
Theta Probability/Permutations/Combinations Solutions Page 1

1. C Combination since a group is being chosen, so the answer is ${}_{15}C_2$
2. B Combination since a group is being chosen, so the answer is $({}_{25}C_3)({}_{10}C_2)$ = 103500
3. A Permutation since changing order creates a different license plate, so answer is 10^2*26^3 or 1757600
4. A $n(n-1)(n-2)/6 = 12n \Rightarrow (n-1)(n-2) = 72 \Rightarrow n^2 - 3n + 2 = 72 \Rightarrow n^2 - 3n - 70 = 0 \Rightarrow$ $(n-10)(n+3) = 0 \Rightarrow n = 10$ only; ${}_{10}P_3$ = 720
5. C $\dfrac{1-P(A)}{P(A)} = \dfrac{9}{7} \Rightarrow 7 - 7P(A) = 9P(A) \Rightarrow 7 = 16P(A) \Rightarrow P(A) = \dfrac{7}{16}$
6. D Permutation with repeated elements; 12!/(3!2!2!) = 19958400
7. D Number of diagonals = Number of line segments made choosing endpoints from a set of n points subtract n since the sides are not diagonals $\Rightarrow {}_{10}C_3$ - 10 = 45 – 10 = 35
8. D P(at most 16) = 1 – P(sum is 17 or 18); a sum of 17 can occur with 2 6's and 1 5 for a total of 3 ways; a sum of 18 can occur only if all faces are 18 for 1 way $\Rightarrow$ 1 – 4/216 = 53/54
9. E 1 digit: 3 ways (3, 5, or 7)
   2 digits: 3*3 = 9 (last digit has three options, which leaves 3 options for the first digit)
   3 digits: 3*2*3 = 18 (3 options for last digit, then 3 for first digit and 2 for second)
   4 digits: 3*2*1*3 = 18 (3 options for last digits, 3! to do remaining digits)
   $\Rightarrow$ 3 + 9 + 18 + 18 = 48
10. B Set with n elements has $2^n - 1$ proper subsets $\Rightarrow 2^6 - 1$ = 63
11. E To find the constant term, find p and q so that $(x^p)(x^{-2q}) = 1$ and p + q = 9. p – 2q = 0 and p + q = 9 means p = 6 and q = 3, so evaluate $\binom{9}{3}(x)^6\left(-\dfrac{2}{x^2}\right)^3$ = -672
12. E 2010 = 2*3*5*67, so the number of positive integral factors is 2*2*2*2 = 16; factors of 2010 that are multiples of 2 are 2, 2*3, 2*5, 2*67, 2*3*5, 2*5*67, 2*3*67, and 2*3*5*67 or 8 factors $\Rightarrow$ p(factor of 2 is chosen) = 8/16 or ½
13. D Create a table to summarize the data with given information shaded; also use the fact that 1/5(10) = 2, so there are 2 black male dogs

		Female	Male	Totals
P(non black female)	Black	1	2	3
=4/10=2/5	Not Black	4	3	7
		5	5	

14. C a = 5 + 7 = 12; ${}_{12}P_2$ = 132 $\Rightarrow$ a + b = 12 + 132 = 144
15. D Let A = smallest radius, B = middle radius, C = largest radius; find A/B. Since the three regions of the target are congruent, $\pi A^2 = \pi C^2/3$, $\pi A^2 = \pi(B^2 - A^2)$, and $\pi A^2 = \pi(C^2 - B^2)$. The first equation gives $A\sqrt{3} = C$; sub this into equation 3 and $A^2 = 3A^2 - B^2$, so $B^2 = 2A^2 \Rightarrow B = A\sqrt{2}$ and A/B = $1/\sqrt{2}$
16. A Use geometry; graph A on the x-axis and B on the y-axis; A and B must satisfy the inequality x + y < 2, and the possibilities for A and B form a square with vertices (0,0), (4,0), (0,4), and (4,4) for a possible area of 16. The area enclosed by the inequality is ½*2*2 or 2, so the P(sum is less than 2) = 2/16 or 1/8
17. D Probability of drawing a 3 is $\dfrac{4}{52}$, probability of drawing a king is $\dfrac{4}{52}$. $\dfrac{4}{52} + \dfrac{4}{52} = \dfrac{2}{13}$.
18. A If the number of sides of the polygon is a factor of 360, the measure of the exterior and thus interior angle will be integral; factors of 360 between 3 and 20 are 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, and 20 so of the 18 possible polygons, 11 will have integral degree measures and 7 will not: $\dfrac{7}{20}$
19. C Since the Rook counts as any suit, there are 12 possible cards per suit; the number of hands with all 10 cards in the same suit = (# suits)*(# hands of 10 chosen from 12 different denominations) = $4*{}_{12}C_{10}$ = 264
20. D Each base is occupied or unoccupied, so the number of configurations is 2*2*2 or 8
21. A Let A = severe damage and B = hurricane hits; need P(A∩B) so use conditional probability: P(A|B) = (A∩B)/P(B); P(B) = .40 and P(A|B) = .10 $\Rightarrow$ .10 = P(A∩B)/.40 $\Rightarrow$ P(A∩B) = 4%
22. B P(at least three digits in base 6) = P(# > 35 in base 10) = 1 – P(# ≤ 35 in base 10) = 1 – 35/10000 or 1993/2000
23. D 2 cases: both G's are used or only 1 G is used $\Rightarrow$ choose the letters in each case and arrange them; if only 1 G is used, even though there are two ways to choose the G, the letters arranged will be the same set and so it is not necessary to multiply by ${}_2C_1$ $({}_2C_2*{}_5C_4)$*6!/2! + $({}_5C_5)$*6! = 2520

24\. C L has 20 points: (0, ±25), (±25, 0), (±7, ±24), (±24, ±7), (±15, ±20), (±20, ±15). Of these, if the set of 4 chosen is one of the last four groups of points, then the quadrilateral will be a rectangle which is the only type of parallelogram that can be inscribed in a circle ⇒ $4/{}_{20}C_4$ or 4/4845

25\. B (# ways to arrange consonants)*(# ways to arrange vowels)*(# ways to place the vowels—at beginning or end, or between a consecutive pair of consonants) = 5!*2!*6 = 1440

26\. C For distinct digits, there are 10! ways of selecting the ID #s. There are only 2 ways to have the numbers increasing and consecutive, namely 0…8 and 1…9. $\frac{2}{10!}$

27\. A In order for a locker to be left open, it must have been touched an odd number of times. For each factor of the locker number, there is 1 touch and so the lockers left open will have an odd number of integral factors. The only way for this to happen is if the locker number is a perfect square, and so it is necessary only to count the number of perfect squares less than 500. Since $23^2 = 529$, there are 22 lockers left open.

28\. D Let W be the event "woman chosen" and J be the event "junior chosen." Find

$P(W \cup J)$. P(W)=$\frac{640}{1200}=\frac{8}{15}$; P(J)=$\frac{360}{1200}=\frac{3}{10}$; $P(W \cap J)=\frac{200}{1200}=\frac{1}{6}$;

$$P(W \cup J) = P(W) + P(J) - P(W \cap J) = \frac{8}{15} + \frac{3}{10} - \frac{1}{6} = \frac{2}{3}$$

29\. C Drawing two chips simultaneously is equivalent to drawing a chip and then drawing another chip without replacing the first one. P(red, then blue) or P(blue, then red)= P(red, then blue)+ P(blue, then red)= $\frac{4}{15}+\frac{4}{15}=\frac{8}{15}$

30\. D To end on the fourth question, the loser must be exactly 2 questions behind the winner at the end of the fourth question. The game would have stopped earlier than round 4 if the loser gets 3 or more questions behind the winner. Consider the following cases: A) winner has 4 correct at the end of 4 questions, B) winner has 3 correct answers at the end of 4 questions, and C) winner has 2 correct answers at the end of 4 questions. One correct answer at the end of the 4 questions is not possible even because the other player would still have an opportunity to tie depending on the results of the fifth question.

Case A: If the winner has 4 correct at the end of question 4, the loser must have had 2 correct answers in the first 3 questions and the loser must miss question 4. The two scenarios are Big Bird wins and Barney wins.

Big Bird wins = Big Bird gets 4 correct and Barney gets 2 correct in 1 – 3 and misses 4
$(3/4)^4$ * $(3*(3/5)^2*(2/5))*2/5$ = 8748/160000

Barney wins = Barney gets 4 correct and Big Bird gets 2 correct in 1 – 3 and misses 4
$(3/5)^4$ * $(3*(3/4)^2*(1/4))*1/4$ = 2187/160000

Case B: The winner must get 2 of the first three questions correct and the fourth question must be answered correctly while the loser gets 1 correct of the first three questions and incorrectly answers the fourth question.

Big Bird wins = Big Bird gets 2 correct in 1 – 3 and #4 and Barney gets one of the first 3 and misses #4
$(3*(3/4)^2*(1/4))*3/4$ * $(3*(3/5)*(2/5)^2)*2/5$ = 5832/160000

Barney wins = Barney gets 2 correct in 1 – 3 and #4 and Big Bird gets one of the first 3 and misses #4
$(3*(3/5)^2*(2/5))*3/5$ * $(3*(3/4)*(1/4)^2)*1/4$ = 1458/160000

Case C: The winner gets 1 correct of the first 3 and the fourth question correct, and the loser gets none of the four questions correct.

Big Bird wins = Big Bird gets 1 correct in first 3 and #4 and Barney gets none of the questions correct
$(3*(3/4)*(1/4)^2)*3/4$ * $(2/5)^4$ = 432/160000

Barney wins = Barney gets 1 correct in first 3 and #4 and Big Bird gets none of the questions correct
$(3*(3/5)*(2/5)^2)*3/5$ * $(1/4)^4$ = 108/1600000

Since either of the six scenarios can occur the probability that the game ends in exactly four rounds is the sum of the six probabilities above, or 3753/32000