Question #1
Theta Bowl – Mu Alpha Theta National Convention 2003

Given: Points $A(3, -4)$ and $B(-5, -3)$

The line perpendicular to $\overline{AB}$ at point B intersects the line with equation $2x - y = 5$ at the point (C, D).

Find: $C + D$

Question #2
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Let A = the value of x if $\sqrt{x + \sqrt{x + \sqrt{x + \cdots}}} = 10$

Let B = the positive value of y if $111_{\text{base } y} = 43_{\text{base } 10}$

Let C = the maximum number of negative real zeros of the function $f(x) = x^4 + 5x^3 - 3x^2 - x - 1$

Let D = the value of $f(4)$ if
$f(1) = 5$
$f(2) = -4$
$f(n) = 3 \cdot f(n-1) + 2 \cdot f(n-2)$, for $n > 2$

Find: $\dfrac{A}{BC} + D$

Question #3
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The statements below have been assigned various point values, given in parentheses at the beginning of each statement. Find the sum of the point values of all statements that are true.

(10) For all real values of x, $\sqrt{x^6} = x^3$.

(3) The graph of $f(x) = \dfrac{x^2 - 4}{x^2 + x - 6}$ has two vertical asymptotes.

(–4) For all real values of x, the minimum value of $x^2 - 4x + 5$ is 2.

(2) $3xy^3z^3 + 6\sqrt{x}$ is a polynomial of degree 7.

(6) $i^{2000} + i^{2001} + i^{2002} + i^{2003} = 0$ $\left(\text{where } i = \sqrt{-1}\right)$

(–5) All real numbers are complex.

(–1) If $f(x) = \log_b x$ ($b > 0$, $b \neq 1$) and c and d are real numbers such that $d > c$, then $f(d) > f(c)$.

(5) The graph of a cubic function must have at least one x-intercept.

(7) ${}_{2003}C_{2000} = {}_{2003}C_3$

(4) The inverse of $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ is $\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}$.

Question #4
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Let A = the number of square units in the intersection of the graphs of
$$\begin{cases} x^2 + y^2 \le 64 \\ y \ge |x| \end{cases}$$

Let B = the number of square units in the area enclosed by the graph of $16x^2 + 9y^2 = 144$

Let C = $\log\left(\frac{1}{2}\right) + \log\left(\frac{2}{3}\right) + \log\left(\frac{3}{4}\right) + \cdots + \log\left(\frac{999}{1000}\right)$

Let D = $(\log_2 3)(\log_3 4)(\log_4 5)\cdots(\log_{63} 64)$

Find: $\frac{ACD}{B}$

Question #5
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Let A = the length of the median of a trapezoid given that the longer base of the trapezoid has length 18 and the length of the median is 6 less than twice the length of the shorter base

Let B = the number of inches in the perimeter of a rhombus whose diagonals have lengths 48 inches and 20 inches

Find: $A + B$

Question #6
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A point moves in a plane such that its distance from the point $(3, -2)$ is $\frac{2}{3}$ its distance from the point $(1, 2)$.
If the equation of this locus of points is written in the form

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

(A, C, D, E, and F are relatively prime integers and $A > 0$), find $A + C + D + E + F$

Question #7
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Let A = the value of x that satisfies the equation $4^{\log_2 x} = 25$

Let B = the numerical value of ${}_8C_2$

Let C = the sum of all real values of x that satisfy the equation $x^3 - x^2 - 24x - 36 = 0$

Let D = $g[f(-1)]$ if $f(x) = 5x - 3$ and $g(x) = -x^2 - 4x + 2$

Find: $\frac{D}{A} + \frac{B}{C}$

Question #8

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Let P = the value of x if $f(x) = \log_3 (x + 1)$ and $f(2x) + f(x) = 1$

Let Q = the value of k that makes the graph of $3x + ky = 5$ perpendicular to the graph of $2x - y = -2$

Let P and Q be the roots of a quadratic equation. If this equation is written in the form $ax^2 + bx + c = 0$, where a, b, and c are relatively prime integers and $a > 0$, find the numerical value of $ab + c$.

Question #9

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Let A = the value of y when $x = 9$ and $z = 2$, given that y varies directly as x and inversely as the square of z, and $y = 2$ when $x = 3$ and $z = 4$

Let B = the sum of the first 100 positive odd integers

Let C = the number of square units in the area of a triangle with vertices $(2, 1)$, $(-1, 7)$ and $(8, 2)$

Let D = the numerical value of $(1 + i)^{20}$ $\left(\text{where } i = \sqrt{-1}\right)$

Find: $$\frac{B}{A + D} + 2C$$

Question #10

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Let A = the arithmetic mean of 30 and 120

Let B = the positive geometric mean of 30 and 120

Let C = the harmonic mean of 30 and 120

Find: $A + B + C$

Question #11

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Let A = the value of $-(-x - x^x)^x$, given that $x = -1$

Let B = the number of positive 3-digit integers that are multiples of 5 that can be formed using only the digits 0, 1, 2, 3, 4, 5, 6 with repetition of digits allowed

Let C = the 4th term of a geometric sequence whose first three terms are $n - 2$, $n + 4$, $5n + 2$, where n is a positive integer

Let D = the sum of all positive integers between 35 and 180 that are evenly divisible by 8

Find: $$AB + \frac{D}{C}$$

Question #12
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Let A = the length of the radius of the circle with center $(4, 11)$ and tangent to the line with equation $y = \frac{3}{5}x + \frac{9}{5}$

Let B = the number of digits in the product $(2^{63})(5^{64})$

Let C = the units digit of 2003^{2003}

Let D = the sum of all real values of x that satisfy the equation $4x^{\frac{2}{3}} + x^{\frac{1}{3}} - 5 = 0$

Find: $A^2 + BD + C$

Question #13
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Given: $\log 2 = a$

Solve the given equation for x in terms of a:

$$\left(\frac{1}{8}\right)^{3x+1} = \left(\frac{1}{100}\right)^{2x-1}$$

Question #14
Theta Bowl – Mu Alpha Theta National Convention 2003

Given: $M = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

Find the sum of the entries in $M^2 - M^T + M \cdot M^{-1}$.

Queston #15
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Find the sum of all integral values of x that satisfy the given inequality:

$$x^2 - 5x - 66 \leq 0$$

Theta Bowl Solutions
2003 National Mu Alpha Theta Convention

(1) $m_{\overline{AB}} = \frac{-3-(-4)}{-5-3} = -\frac{1}{8}$

$m_{\perp} = 8$

$y+3 = 8(x+5)$

$y = 2x-5$

$2x-5+3 = 8x+40$

$-42 = 6x$

$x = -7$

$y = 2(-7)-5 = -19$

$(C, D) = (-7, -19)$

$C + D = \boxed{-26}$

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(2) A: $\sqrt{x+10} = 10$

$x+10 = 100$

$x = 90$

B: $y^2+y+1 = 43$

$y^2+y-42 = 0$

$(y+7)(y-6) = 0$

positive $\Rightarrow y = 6$

C: $f(-x) = x^4-5x^3-3x^2+x-1$

3 sign reversals

D: $f(1) = 5$

$f(2) = -4$

$f(3) = 3(-4)+2(5) = -2$

$f(4) = 3(-2)+2(-4) = -14$

$\frac{A}{BC} + D = \frac{90}{(6)(3)} - 14 = \boxed{-9}$

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(3) $\sqrt{x^6} = x^3$ only if $x \geq 0$ (false)

$f(x) = \frac{(x+2)(x-2)}{(x+3)(x-2)}$ has one vertical asymptote at $x=-3$ (false)

The minimum value of x^2-4x+5 is $2^2-4(2)+5 = 1$ (false)

$3xy^3z^3 + 6\sqrt{x}$ is not a polynomial (false)

$i^{2000}+i^{2001}+i^{2002}+i^{2003} = 1+i+(-1)+(-i) = 0$

(6) ☆(true)

{Complex Numbers} = {Reals} ∪ {Imaginaries}

(−5) ☆(true)

$f(d) > f(c)$ only if $b>1$ (false)

A polynomial function of odd degree must have at least one x-intercept

(5) ☆ true

${}_{2003}C_{2000} = {}_{2003}C_3$

(7) ☆ true

the inverse of $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ is $\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$.

(false)

$6+(-5)+5+7 = \boxed{13}$

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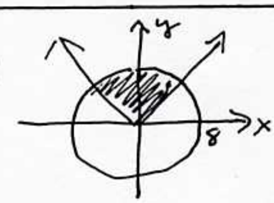
(4) A:



$\frac{90°}{360°}(64\pi) = 16\pi$

B: $\frac{x^2}{9} + \frac{y^2}{16} = 1$ $\quad \pi(ab) = \pi(4)(3) = 12\pi$

C: $\log_{10}\left(\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdots\frac{999}{1000}\right) = \log\left(\frac{1}{1000}\right) = -3$

D: $\frac{\log 3}{\log 2}\cdot\frac{\log 4}{\log 3}\cdot\frac{\log 5}{\log 4}\cdots\frac{\log 64}{\log 63} = \log_2 64 = 6$

$\frac{ACD}{B} = \frac{(16\pi)(-3)(6)}{12\pi} = \boxed{-24}$

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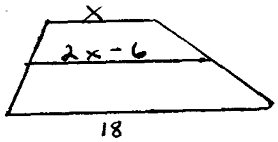
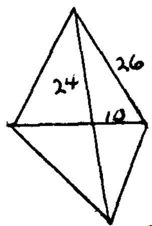
(5) A:



$$\frac{x+18}{2} = 2x-6$$
$$x+18 = 4x-12$$
$$30 = 3x$$
$$10 = x$$
$$2x-6 = 14$$

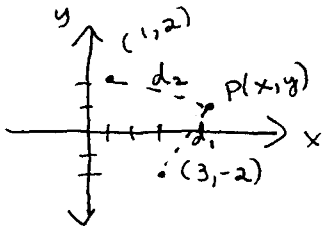
B:



$$26(4) = 104$$

$$A + B = 14 + 104 = \boxed{118}$$

(6)



$$d_1 = \frac{2}{3} d_2$$
$$3d_1 = 2d_2$$
$$3\sqrt{(x-3)^2+(y+2)^2} = 2\sqrt{(x-1)^2+(y-2)^2}$$
$$9\left[x^2-6x+9+y^2+4y+4\right] = 4\left[x^2-2x+1+y^2-4y+4\right]$$
$$9x^2-54x+9y^2+36y+117 = 4x^2-8x+4y^2-16y+20$$
$$5x^2+5y^2-46x+52y+97=0$$
$$A+C+D+E+F = \boxed{113}$$

(7) A: $4^{\log_2 x} = 25$

$$2^{2\log_2 x} = 25$$
$$2^{\log_2 x^2} = 25$$
$$x^2 = 25$$
$x = 5$ (reject $x=-5$)

C:

$$\begin{array}{r|rrrr} -2 & 1 & -1 & -24 & -36 \\ & & -2 & 6 & 36 \\ \hline & 1 & -3 & -18 & 0 \end{array}$$

$$(x+2)(x^2-3x-18)=0$$
$$(x+2)(x-6)(x+3)=0$$
$$\{-2, 6, -3\}$$
sum $= 1$

B: $_8C_2 = \frac{8!}{2!6!} = \frac{8 \cdot 7}{2} = 28$

D: $f(-1) = 5(-1)-3$

$$f(-1) = -8$$
$$g(-8) = -(-8)^2-4(-8)+2 = -30$$

$$\frac{D}{A}+\frac{B}{C} = \frac{-30}{5}+\frac{28}{1}$$

$$\boxed{22}$$

(8) P: $f(2x) = \log_3(2x+1)$

$$\log_3(2x+1)+\log_3(x+1) = 1$$
$$\log_3(2x^2+3x+1) = 1$$
$$2x^2+3x+1 = 3$$
$$2x^2+3x-2 = 0$$
$$(2x-1)(x+2)=0$$

$x = 1/2$ or $x=-2$ (reject

Q: $\frac{-3}{K} = \frac{-1}{2} \Rightarrow K = 6$

$\frac{1}{2}+6 = \frac{13}{2}$ | $\frac{1}{2}\cdot 6 = 3 = \frac{6}{2}$

$$2x^2-13x+6=0 \Rightarrow ab+c = \boxed{-20}$$

(9) A: $y = \frac{kx}{z^2} \Rightarrow K = \frac{yz^2}{x} \Rightarrow \frac{y(2)^2}{9} = \frac{2(4)^2}{3}$

$y = 24$

B: Sum $= (100)^2 = 10{,}000$

C:

21
-17 14
-1 82 -2
56 21 8
4

$$\frac{59-20}{2} = \frac{39}{2}$$

D: $(1+i)^{20} = (2i)^{10} = 1024 i^{10} = -1024$

$$\frac{B}{A+D}+2C = \frac{10{,}000}{-1{,}000}+39 = \boxed{29}$$

(10) A: $\frac{30+120}{2} = 75$

B: $+\sqrt{30(120)} = 60$

C: $\frac{2(30)(120)}{30+120} = 48$

$A+B+C =$

$$\boxed{183}$$

(11) A: $-(-(-1)-(-1)^{-1})^{-1} = -(1+1)^{-1} = -\frac{1}{2}$

B: $6 \cdot 7 \cdot 2 = 84$ (first digit ≠ 0 and last digit 0 or

C: $\frac{5n+2}{n+4} = \frac{n+4}{n-2} \Rightarrow 5n^2-8n-4 = n^2+8n+16$

$$4n^2-16n-20=0 \Rightarrow n^2-4n-5=0$$
$(n-5)(n+1)=0 \Rightarrow n=5$ (reject $n=-1$)

$3, 9, 27 \Rightarrow 4^{th}$ term 81

D: $40+48+\ldots+176$

$$176 = 40+(n-1)8$$
$$136 = (n-1)8 \Rightarrow n = 18$$
$$S_{18} = \frac{18}{2}(40+176) = 1944$$

$$AB+\frac{D}{C} = \left(-\frac{1}{2}\right)(84)+\frac{1944}{81}$$

$$-42+24 = \boxed{-18}$$

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(12) A: $y = \frac{3}{5}x + \frac{9}{5}$

$5y = 3x + 9$

$3x - 5y + 9 = 0$

distance from $(4, 11)$ to $3x - 5y + 9$

$$\frac{|3(4) + (-5)(11) + 9|}{\sqrt{3^2 + (-5)^2}} = \frac{34}{\sqrt{34}} = \sqrt{34}$$

B: $(2 \cdot 5)^{63} \cdot 5 = 5 \cdot 10^{63}$

64 digits

C: $\left.\begin{array}{llll} 3^1 \to & \text{units digit} & & 3 \\ 3^2 \to & \text{"} & \text{"} & 9 \\ 3^3 \to & \text{"} & \text{"} & 7 \\ 3^4 \to & \text{"} & \text{"} & 1 \end{array}\right\} 4$

$\vdots$

$3^{2003} \to$ " " 7

D: Let $y = x^{1/3}$

$4y^2 + y - 5 = 0$

$(4y + 5)(y - 1) = 0$

$y = -\frac{5}{4}$ or $y = 1$

$x^{1/3} = -\frac{5}{4}$ or $x^{1/3} = 1$

$x = \frac{-125}{64}$ or $x = 1$

sum $= \frac{-61}{64}$

$A^2 + BD + C = (\sqrt{34})^2 + (64)\left(\frac{-61}{64}\right) + 7$

$34 - 61 + 7 = \boxed{-20}$

(13) $\left(\frac{1}{8}\right)^{3x+1} = \left(\frac{1}{100}\right)^{2x-1}$

$(3x+1)\log\left(\frac{1}{8}\right) = (2x-1)\left(\log\left(\frac{1}{100}\right)\right)$

$(3x+1)(-3\log 2) = (2x-1)(-2)$

$-3a(3x+1) = -4x + 2$

$-9ax - 3a = -4x + 2$

$x(4 - 9a) = 2 + 3a$

$x = \boxed{\frac{2+3a}{4-9a}}$

(14) $M^2 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

$M^2 = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 3 \\ 2 & 1 & 5 \end{bmatrix}$

$M^T = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

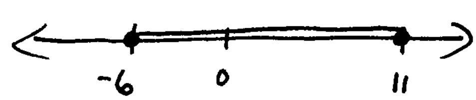
$M \cdot M^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$M^2 - M^T + M \cdot M^{-1} = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & 2 \\ 1 & 0 & 4 \end{bmatrix}$

sum of entries $= \boxed{14}$

(15) $x^2 - 5x - 66 \le 0$

$(x - 11)(x + 6) \le 0$



$-6 + (-5) + \cdots + 7 + 8 + 9 + 10 + 11 = \boxed{45}$