ALPHA ANALYTIC GEOMETRY
FAMAT State Convention 2004

For all questions, E. NOTA means none of the above answers is correct.

1) Find the equation for the line in the *xy*-plane which represents the locus of all points equidistant from the points (1,2) and (3,7).

a) $4x+10y-53=0$ b) $2x+5y-13=0$ c) $10x+5y-37=0$
d) $2x-5y-27=0$ e) NOTA

2) List the eccentricities of the following conic sections from least to greatest:
i)parabola ii) ellipse iii) hyperbola iv) circle

a) $i<ii<iii<iv$ b) $iv<ii<i<iii$ c) $iii<ii<i<iv$ d) $iv<i<ii<iii$ e) NOTA

3) The three cube roots of 1 form a triangle centered about the origin of the complex plane. Find the area of this triangle.

a) $\frac{3\sqrt{3}}{2}$ b) $\frac{3\sqrt{3}}{4}$ c) $\frac{2+\sqrt{3}}{4}$ d) $\frac{3\sqrt{3}}{8}$ e) NOTA

4) Find the area of the convex polygon with vertices located at (-4,5), (-6,2), (-3,-2), (2,-1), (3,4), (1,6), and (-2,7).

a) $\frac{109}{2}$ b) $\frac{49}{2}$ c) $\frac{79}{2}$ d) 15 e) NOTA

5) A crazy heptagon with vertices located at (-6,2), (-3,-2), (2,-3), (4,-1), (3,4), (1,6), and (-2,7) has been mysteriously revolved around the line with equation $7x-7y-70=0$. The area of the heptagon is $\frac{131}{2}$. A visit from the Calculus Fairy reveals that the volume of such a figure can be found by the formula $V=2\pi\cdot MN$, where M is the distance from the centroid of the heptagon to the line and N is the area of the heptagon. Find the volume of this polygonal taurus.

a) 131π b) $4108\sqrt{2}\pi$ c) 100π d) $786\sqrt{2}\pi$ e) NOTA

6) Given vectors $\mathbf{u}=\langle 5,6\rangle$ and $\mathbf{v}=\langle 10,0\rangle$, find $\text{proj}_{\mathbf{v}}\mathbf{u}$, that is, the projection of **u** onto **v**.

a) $\langle 2.5,3\rangle$ b) $\langle 5,0\rangle$ c) $\langle 10,12\rangle$ d) $\langle 5,1\rangle$ e) NOTA

7) Let x be one of the roots of the equation $y^3+27x^2y+9y^2x+27x^3=1$. Find $|x|$.

a) $\frac{1}{3}$ b) 1 c) –1 d) $\frac{1}{6}$ e) NOTA

8) Appolonius of Perga, working for a dog leash manufacturer, designs a leash permitting the canine to travel only as follows: he must stay equidistant between the fence ($y = 1/2$) and a stake in the ground, located at (3,4). Find the locus of points (on the ground) on which the dog can travel. (hint: the test-taker may note the ineffectiveness of the leash, which allows the dog to travel infinitely far away from the fence.)

a) $(x-3)^2 = 7\left(y-\frac{9}{4}\right)$ b) $(x-3)^2 = -7\left(y+\frac{9}{4}\right)$ c) $(x+3)^2 = 7\left(y+\frac{9}{4}\right)$

d) $\left(x-\frac{9}{4}\right) = -7(y-3)^2$ e) NOTA

9) Find the equation of a circle with radius 4 and centered at (1,5).

a) $x^2+y^2+2x+10y-22=0$ b) $x^2+y^2+2x+10y-10=0$

c) $x^2+y^2-2x-10y+10=0$ d) $x^2+y^2-2x-10y+22=0$ e) NOTA

10) A double-napped cone is represented by the equation $x^2+y^2-z^2=0$. Find the volume enclosed by the double-napped cone and the planes $z=-6$ and $z=9$.

a) 315π b) 117π c) 81π d) 243π e) NOTA

11) How many points of intersection are there between the graphs represented in polar coordinates by $r=3$ and $r=3+6\cos(\theta)$.

a) 2 b) 3 c) 0 d) 4 e) NOTA

12) Appolonius has constructed another dog leash. This contraption consists of two stakes on the ground, located at (-3,0) and (3,0), and a rope of length 8 attached at both ends to the stakes. The dog is permitted to slide along the rope as he pleases, leaving slack in the line if he wishes. What area of the yard can the dog cover?

a) $4\sqrt{7}\pi$ b) 16π c) 20π d) $12\sqrt{3}\pi$ e) NOTA

13) Locate the x-coordinates of all of the discontinuities of the equation $y=\frac{\sin^2 x+\cos^2 x+2\sin x\cos x}{\sin^2 x-\cos^2 x}$ in terms of the integer, n.

a) $x=n+\frac{\pi}{4}$ b) $x=n-\frac{\pi}{4}$ c) $x=n\pm\frac{\pi}{4}$ d) $x=n-\frac{3\pi}{4}$ e) NOTA

14) Let there be two lines, $L_a: y=4x$ and $L_b: 4y=x$. Find the set of all points in the same plane as the lines such that the distance between any point and L_a is twice the distance between the point and L_b.

a) $y=\frac{-2}{7}x$ b) $y=x^2+4$ c) $y=\frac{2}{3}x$ d) both a) and c) e) NOTA

15) Find the volume of the parallelopiped described by the vectors $\langle -1,2,1\rangle$, $\langle 0,0,3\rangle$, and $\langle 2,-4,-2\rangle$.

a) 4 b) $\frac{5}{2}$ c) 6 d) $\frac{15}{2}$ e) NOTA

16) Find the equation of a circle with center (-2,3) that is tangent to the line $x - y = 7$.

a) $x^2 + y^2 - 4x + 6y + 57 = 0$ b) $x^2 + y^2 + 4x - 6y - 59 = 0$
c) $x^2 + y^2 + 2x - 3y - 118 = 0$ d) $x^2 + y^2 + 4x - 6y - 34 = 0$ e) NOTA

17) A parabola has focus $(0, p)$ and directrix $y = -p$. If $p \neq 0$, which of the following is the equation for this parabola?

a) $x^2 = 2py$ b) $x^2 = 8py$ c) $y^2 = 2px$ d) $x^2 = 4py$ e) NOTA

18) Find the polar equation of a line passing through the point $\left(1,-\sqrt{3}\right)$ and having slope $-\sqrt{3}$.

a) $\theta = \frac{\pi}{3}$ b) $\theta = \frac{-\pi}{6}$ c) $\theta = \frac{-\pi}{4}$ d) $r = \frac{-\theta}{3}$ e) NOTA

19) What is the measure, to the nearest degree, of the largest angle possible formed by the intersection of two diagonals of a cube.

a) 90 b) 109 c) 120 d) 103 e) NOTA

20) If the difference in area between two circles is 10π, and the sum the radii of the two circles is 5, find the radius of the larger of the two circles.

a) 4 b) 2.75 c) 3.5 d) $\frac{9}{\pi}$ e) NOTA

21) Find the cosine of the angle between the two vectors $\langle -2,3\rangle$ and $\langle 1,7\rangle$.

a) $\frac{19\sqrt{26}}{130}$ b) $\frac{19\sqrt{26}}{26}$ c) $\frac{21\sqrt{26}}{26}$ d) $\frac{23\sqrt{26}}{130}$ e) NOTA

22) What is the slope of the line that passes through the intersection of the graphs $y = \log_4 x$ and $y = 4^{x-1} - 1$?

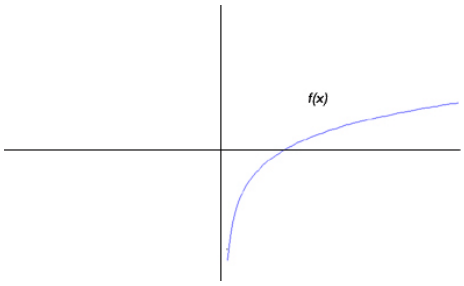
a) $\frac{1}{4}$ b) 1 c) $\frac{1}{2}$ d) 2 e) NOTA

23) The equation $2x^2 - 2y^2 - x + 11y - 15 = 0$ can be best described as a (an):

a) hyperbola b) ellipse c) pair of lines d) single point e) NOTA

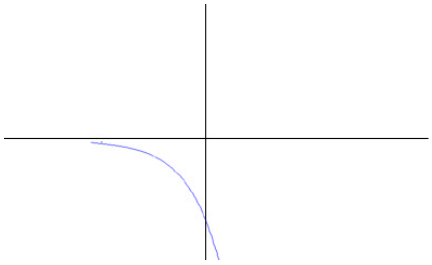
24).

If the following graph represents $f(x)$:

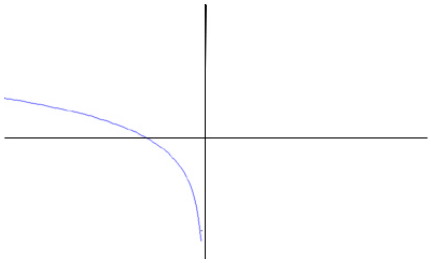


then which of the following best represents the graph of $f^{-1}(-x)$?

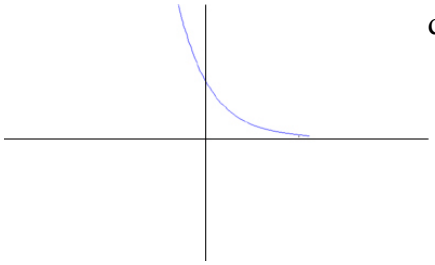
a)



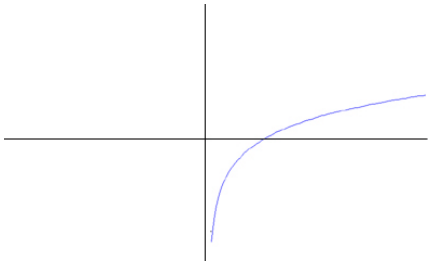
b)



c)



d)



e) NOTA

25) A triangle with sides of length 4, 4, and 6 is inscribed in a circle. Find the area of that circle.

a) $\frac{64\pi}{7}$ b) $\frac{16\pi}{7}$ c) $\frac{9\pi}{7}$ d) 9π e) NOTA

26) Find the distance between the two points (to the nearest tenth) represented in polar coordinates by $\langle 2,35° \rangle$ and $\langle 5,215° \rangle$.

a) 4.6 b) 6.5 c) 8.8 d) 7.0 e) NOTA

27) Which of the following is a polar representation of a parabola?

a) $r = \frac{6\cos\theta}{\sin^2\theta}$ b) $r = \frac{4}{1-3\sin\theta}$ c) $r = \frac{12}{3-6\sin\theta}$ d) $r = \frac{3}{1+2\cos\theta}$ e) NOTA

28) In a particular ellipse, if the length of the major axis, A_{major}, and the length of the minor axis, A_{minor}, have the relationship $\frac{A_{\text{major}}}{A_{\text{minor}}} = 3$, and the area of the ellipse is 48π, then the length of the latus rectum of the ellipse is equal to which of the following?

a) 72 b) $\frac{72}{5}$ c) $\frac{8}{3}$ d) 5 e) NOTA

29) Which of the following is the parametric form of a line crossing the points (2,-3) and (4,7)?

a) $\begin{aligned} x &= t \\ y &= 5t + 13 \end{aligned}$ b) $\begin{aligned} x &= \frac{t}{5} \\ y &= t - 13 \end{aligned}$ c) $\begin{aligned} x &= \frac{t}{2} \\ y &= t - 7 \end{aligned}$ d) $\begin{aligned} x &= 10t \\ y &= 4t + 5 \end{aligned}$ e) NOTA

30) If the circumference of an ellipse can be approximated by the formula $C = \pi\left(3a + 3b - \sqrt{(a+3b)(b+3a)}\right)$, where *a* and *b* are the lengths of the major and minor axes, respectively, then find the area of a circle (to the nearest tenth) that has the same circumference of an ellipse with $a = 7$ and $b = 4$.

a) 26.9 b) 98.6 c) 41.3 d) 61.3 e) NOTA