

ALPHA LOGS AND EXPONENTS

FAMAT State Convention 2004

For all questions, E. NOTA means none of the above answers is correct. Let $i = \sqrt{-1}$.

1. Solve the following equation for x and round to three significant digits.

$$e^{2x} = 3$$

- A) 0.176 B) 0.239 C) 0.405 D) 0.549 E) NOTA

2. Which of the following is equivalent to $\log_a b$?

- A) $\log_b a$ B) $\log_{\frac{1}{a}} \frac{1}{b}$ C) $\log_{2a}(2b)$ D) $\log_{a+1}(b+1)$ E) NOTA

3. Let $f(x)$ be an exponential function of the form $f(x) = ab^x$ where a and b are constants. Given that $f(0) = 2$ and $f(1) = 6$, find $f(5)$.

- A) 96 B) 486 C) 842 D) 1,458 E) NOTA

4. Round the decimal number equivalent to $\log_{11} 18$ to three significant digits.

- A) .830 B) 1.04 C) 1.21 D) 1.26 E) NOTA

5. Round to the nearest integer:

$$\log_{10}(2^{1000})$$

- A) 3 B) 300 C) 301 D) 600 E) NOTA

6. How many digits are in 7^{89} ? Hint: take the logarithm.

- A) 67 B) 68 C) 75 D) 76 E) NOTA

7. Find the sum of the solutions of the following equation. Hint: use a substitution.

$$2e^{2x} - 4e^x + 1 = 0$$

- A) $\ln\left(\frac{1}{2}\right)$ B) $\ln(2)$ C) 1 D) 2 E) NOTA

8. Given: $f(n) = n * f\left(\frac{n}{2}\right)$, $f(1) = 1$

Find: $\log_2(f(256))$

- A) 7 B) 8 C) 28 D) 36 E) NOTA

9. Which of the following is equivalent to $e^{5ix} + e^{-5ix}$

- A) $\cos(5x)$ B) $\cosh(5x)$ C) $\sin(5x)$ D) $\sinh(5x)$ E) NOTA

10. Simplify:

$$\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{10}$$

- A) -1 B) $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$ C) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$ D) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ E) NOTA

11. Find the product of the solutions of the following equation where $x > 0$.

$$16x - (2x)^{\log_2 x} = 0$$

- A) 1 B) 2 C) 4 D) 8 E) NOTA

12. Round to the nearest integer:

$$\sum_{n=1}^{10} \log_{10} n$$

- A) 1 B) 5 C) 6 D) 7 E) NOTA

13. If you express $\sqrt{35000}$ in the form $a\sqrt{b}$ where a is the largest possible number such that a and b are integers, what is the value of $a + b$?

- A) 57 B) 64 C) 72 D) 81 E) NOTA

14. Let $a = \log_{12} 5$ and $b = \log_{25} 3$. If you express $\log_5 2$ as a simplified fraction in terms of a and b , what is the denominator of the expression?

- A) $2ab$ B) $1-2ab$ C) $a+b$ D) $2a$ E) NOTA

15. Simplify:

$$(2^3)^2$$

- A) 2^2 B) 2^5 C) 2^6 D) 2^9 E) NOTA

16. In order for a number to be stored by a computer for floating-point arithmetic, the number must be expressed in the form $m \cdot 2^e$ where e is the largest possible integer such that $m \geq 1$. What is the exponent, e , for 1,576?

- A) 8 B) 9 C) 10 D) 11 E) NOTA

17. What is the coefficient of x^7 in the expansion of $(2x - 3)^{10}$?

- A) 3,456 B) 262,440 C) 414,720 D) 2,099,520 E) NOTA

18. If $x \in \left\{ e^{\frac{\pi i}{3}}, e^{\frac{2\pi i}{3}}, e^{\pi i}, e^{\frac{4\pi i}{3}}, e^{\frac{5\pi i}{3}}, e^{2\pi i} \right\}$ and $x^n = 1$ for all values of x , what is the smallest such positive value of n ?

- A) 3 B) 6 C) 9 D) 12 E) NOTA

19. How many of the following are rational multiples of $\log_2 5$?

- I. $\log_5 2$
 II. $\log_8 5$
 III. $\log_{0.5} 0.2$
 IV. $\log_2 25$

- A) 1 B) 2 C) 3 D) 4 E) NOTA

20. Simplify:

$$\log_{(x+2)}(x^3 + 6x^2 + 12x + 8) \quad (\text{Assume } x > -2.)$$

- A) $\log_{(x^2+1)}x$ B) $8\log_{(x^2+1)}x$ C) 3 D) 4 E) NOTA

21. Find the sum of the solutions of the following equation.

$$2^{(x^2)} - 3^{(x+2)} = 0$$

- A) $-3\log_6 2$ B) $\log_3 2$ C) $\log_2 3$ D) $2\log_3 6$ E) NOTA

22. After extensive testing, Willy Wonka found that the diameter of his Everlasting Gobstoppers undergoes exponential decay while an Oompa Loompa sucks on it. The original diameter of an Everlasting Gobstopper was 10 mm and after 1 year of sucking it has shrunk to 5 mm. How many additional years will it take for the Everlasting Gobstopper's diameter to shrink to 1 mm?

- A) $1 - \log_{10} 2$ B) $1 - \log_{10} 5$ C) $\log_5 10 - 1$ D) $\log_2 10 - 1$ E) NOTA

23. Find the domain of $h(g(f(x)))$ where the functions are defined below.

$$f(x) = 10^x \quad g(x) = \sqrt{\frac{x}{2}} \quad h(x) = \log_3(x-1)$$

- A) $x \geq \log_{10} 1$ B) $x \geq \log_{10} 2$ C) $x \geq \log_{10} 3$ D) $x \geq \log_3 2$ E) NOTA

24. If you express $\cos^4(x)$ in the form $a * \cos(4x) + b * \cos(2x) + c$ where a , b , and c are constants, find $a + b + c$? Hint: use $\cos^2(x) = \frac{\cos(2x) + 1}{2}$.

- A) 1 B) $\frac{11}{8}$ C) $\frac{7}{4}$ D) 2 E) NOTA

25. How many of the following are true for all constants x , y , z , p , and q where x is non-zero and p and q are positive?

I. $\frac{x^y}{x^z} = x^{y-z}$

II. $\sqrt[y]{x^y} = x$

III. $p^{\log_p q} = p$

- A) 0 B) 1 C) 2 D) 3 E) NOTA

26. Express $(3 + 4i)^8$ in the form $a + bi$ and find $\frac{b}{a}$ to three significant digits.

- A) -3.43 B) 0.72 C) 1.14 D) 2.15 E) NOTA

27. How many solutions are there to the following equation?

$$x^2 = 2^x$$

- A) 0 B) 1 C) 2 D) 3 E) NOTA

28. If your annual interest rate is 10%, how many years will it take for your money to double if interest is compounded continuously? (Round to 3 significant digits.)

- A) 6.93 B) 7.39 C) 7.62 D) 7.94 E) NOTA

29. Find the sum of the solutions of the following equation.

$$4^{(x+1)} = 2^{(x^2)}$$

- A) -2 B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) 2 E) NOTA

30. Find:

$$\frac{97^{2004}}{5} - \left[\frac{97^{2004}}{5} \right]$$

Note: $[x]$ represents the greatest integer function.

- A) 0 B) 0.2 C) 0.4 D) 0.6 E) NOTA