**Discrete Mathematics States 2004 Solutions**

1. Using logical deduction, the order is Noah, John, Graham **C**

2. The only times when F is true is when $\left(\overline{A}\cdot B\cdot C\right)$ or $\left(A\cdot B\cdot C\right)$ are true, so using sum of products the answer is $F=\left(\overline{A}\cdot B\cdot C\right)+\left(A\cdot B\cdot C\right)$ **A**

3. The negation of (p AND q) is (NOT p OR NOT q). The answer is **B**.

4. The converse of (p implies q) is (q implies p). The answer is **B**.

5. Use DeMorgan's law twice and then the distributive property. **D**

6. The function $f(n)=2n-1$ does the desired mapping. **D**

7. i, iii, and iv are countable. **C**

8. This algorithm has linear complexity since each element must be tested exactly once. **B**

9. $7007 = 7^2*11*13$. **E**

10. m divides the difference between a and b. **A**

11. 6 and 28 are perfect. **A**

12. 3 = 0011 , 7 = 0111 , F = 1111. So 37F = 0011 0111 1111 **D**

13. $\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}*\begin{bmatrix}1&4\\2&5\\3&6\end{bmatrix}=\begin{bmatrix}14&32\\32&77\end{bmatrix}$ **C**

14. Row reduce $\begin{bmatrix}7&-8&5&1&0&0\\-4&5&-3&0&1&0\\1&-1&1&0&0&1\end{bmatrix}$

to get $A^{-1}=\begin{bmatrix}2&3&-1\\1&2&1\\-1&-1&3\end{bmatrix}$ **A**

15. $\Phi(n)$ = n!……. $\Phi(7)$ = 7! = 5040 **E**

16. Using the pigeonhole principle, $\left\lceil\frac{100}{12}\right\rceil=9$ **B**

17. 9nCr3 * 11nCr4 = 27,720 **B**

18. Using the definition of combinations, the expression simplifies to $\binom{n+1}{k}$. **B**

19. $\frac{\text{P(0 in 1st position and 2 consecutive zeros)}}{P(0\ in\ \text{1st position})}=\frac{5/16}{1/2}=\frac{5}{8}$ **D**

20. Use expected value. 0.5(-1)+(1/3)*2 = 0.17 **C**

21. The adjacency matrix shows possible paths from one vertex to another. The correct matrix is in **C**.

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23. An Euler circuit occurs when every vertex has an even degree. **A**

24. A preorder traversal goes in the following order:
    1) Visit root
    2) Visit left subtree
    3) Visit right subtree

The answer is **A**.

25. Only i and ii are true. **E**

26. $a_n$ gives the $n^{th}$ Fibonaccci number, so $a_8$ is 21 **B**

27. C(52,5)*C(47,5)*C(42,5)*C(37,5) = $\frac{52!}{(5!)^4\,32!}$ **C**

28. Using a binomial distribution with p= $2/3$ , the P(4 Heads out of 7 Trials) = 0.26 **D**

29. All statements are true **C**

30. The LCM is $2^4 3^5 7^2$ **B**