**Interschool Test**
**FAMAT State Convention 2004**

1. You may be surprised that the logo for Federal Express contains an arrow. With respect to the logo, which direction does the arrow point?

2. How many miles of interstate are in the state of Florida?

3. The telephone number below can be unscrambled (using the standard letters on each digit of a telephone) to reveal a math word or phrase. What is that word or phrase?

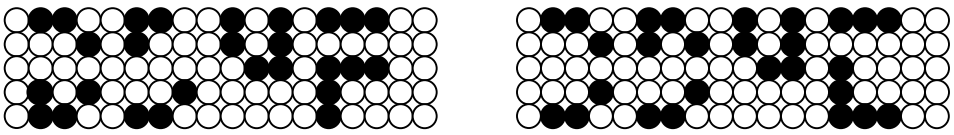
   896-778-2737

4. Same as question 3 but for the phone number:

   732-826-4537

5. The odometer on my car has a place for six digits. How many numbers can be made with those digits such that as you read from left to right, the numbers increase (don't forget to include numbers that start off with a 0!)?

6. How many bills does each player receive at the beginning of a standard game of Monopoly?



7. The lights at a bank thermometer display are not functioning as well as most people would like. It is supposed to show values of the current temperature in degrees Fahrenheit and degrees Celsius. Given the two displays that it shows above, what is/are the possible value(s) of the temperature in degrees F?

8. Colleen works on a rotating shift schedule. The shifts that she could work are: 6 AM to 2 PM, 2 PM to 10 PM, and 10 PM to 6 AM. She gets paid an extra 10%/hr for working at night, that is, between the hours of 6 PM and 6 AM. She gets paid an extra 25%/hr for working anytime on Sunday. She is not allowed to come back to work another shift within 11 hrs of a previous shift, she is not allowed to work the same shift more than 5 times in a row, and she can only work 80 hrs in a 2-week pay period. If her base salary is $20 an hour, and each pay period starts on Sundays at midnights, then what is the range of possible paychecks she can expect to get? Assume that you are looking at the gross pay with no deductions for taxes, etc., and that if she starts the pay period off on the late night shift, the two hours from the night before count toward this paycheck's 80 hrs.

9. Russ has taken out a loan at a bank for $126,424.38 to pay for his new home. It is set up such that he needs to pay $1000/month, starting one month from the date he opened the loan, for 15 years in order to pay off the loan. The interest rate on the loan is 5% compounded monthly. How much more should he pay each month, to the nearest dollar, if he wants to reduce the overall term to 10 years?

10. A weather balloon rises at a constant rate of 5 m/s. The balloon will rise into an environment where the horizontal winds are from the west at a speed that increases at the rate of 5 mph per 1000 ft of elevation, and the surface wind speed is calm. Assume that the balloon moves at the same horizontal speed as the environmental winds. The automatic tracking system for the balloon, located where the balloon is launched, will lose the signal from the balloon when it goes below 5 degrees above the local horizon. How long after the balloon is released, to the nearest minute, will the automatic tracking system lose the balloon? Neglect the curvature effects of the earth.

11. In question 10, at what elevation will the balloon be when the tracking system loses the balloon? Round answer off to the nearest thousand feet.

12. What is the rank of the following matrix?

$$\begin{bmatrix} 1 & -1 & 2 & 0 & 3 \\ 0 & 2 & 1 & 3 & 1 \\ -1 & 1 & 5 & 1 & 0 \\ -1 & 0 & 1 & -1 & -2 \end{bmatrix}$$

13. Proxima Centauri is the closest star, besides the sun of course, to the Earth. It is located at a distance of 4.2 light years from us. How many miles is this? Leave answer in scientific notation with 2 significant figures.

14. The outer radius of curvature of a lune is 1. The inner radius of curvature is 2. The line connecting the 'points' of the lune has length 2. What is the area of the lune?

15. Let a, b, and c be real numbers such that a-8b+12c=9 and 12a+9b-c=8. What is the value of $a^2-b^2+c^2$?

16. Let $\{a_k\}$ be a sequence of integers such that $a_1=1$ and $a_{m+n}= a_m+a_n+mn$, for all positive integers m and n. What is the value of $a_{2004}$?

17. Tax season has just ended for some very busy accountants. In honor of them, here is a fairly easy tax question. Barry is a single guy who made a total adjusted gross income of $20,000 in 2003. He had no deductions he could make besides the standard deduction and the exemption for himself, no credits he could take, and no additional taxes beyond the standard income tax. He had a total of $1,000 withheld from his paychecks during the year. How much does he owe/how much should he receive?

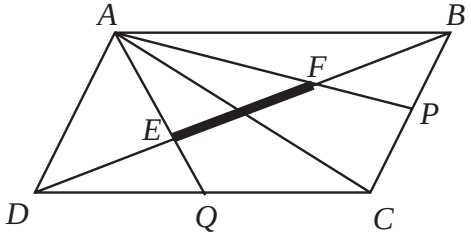
18. Speaking of mid April, we also just passed the 92nd anniversary of the sinking of the Titanic. On what day of the week did she founder?

19. Speaking of calendars and days, how many years does it take for our current calendar system to make an error of one full day in calculating our rotation around the Sun?

20. A certain recursive relation is defined by $x_{n+1} = x_n + x_{n-1}$. What is $\lim_{n\to\infty}\left(x_{n+1}/x_n\right)$.

21. If a, b, and c are distinct roots of $3x^3+0.2x^2-3x-.1=0$, then what is the numerical value of $a^3+b^3+c^3$?

22. In the figure below, ABCD is a parallelogram with P and Q the midpoints of sides BC and CD, respectively. If AB=10, BC=8, and AC=12, then what is the length of EF (the bold segment)?




23. How many digits are in the full expansion of 2004! ?

**Interschool Test**
**FAMAT State Convention 2004**

24. The following quotation is given as a cryptogram. On your answer sheet please write out the quote and who wrote it.

"S XLTK TKLZO RBUSKSWT JMZ EJC KVJK XO TBMT XJO VJQD USPDCKO KB

TKLZO XJKVDXJKSWT JMZ RVSUBTBRVO."

FBVM JZJXT

25. What are the last four digits in the full expansion of $(2004^{2004})^{2004}$?

26. What is the next number in the following sequence:
144, 377, 987, 2584, 6765, _____

27. A given volume of air contains 600 raindrops per cubic meter. Each drop has a diameter of 1 mm and falls at a rate of 4 m/s. What is the rainfall rate, which is the amount of rain that would be measured in one hour? Leave your answer to the nearest hundredth of an inch per hour.

28. Radar reflectivity (Z) can be used to estimate the rainfall rate (R). There are many different equations relating the two with the dependence being on the type of weather expected to cause the rain. One such relation is that $Z=300R^{1.4}$. Given the answer to the previous question, how many dBZ, to the nearest dBZ, would a radar picture show for the given rainfall rate(where dBZ=10logZ)?

29. What is the sum of the following sequence of numbers:
2, -2/3, 41/144, -5/432, 1403/20736, 2135/124416, …

30. What fraction of all five-digit numbers have the property that none of the digits are multiples of three, yet the sum of the digits is a multiple of three? Note: For this question, assume that a five-digit number cannot start with zero and assume that zero is not a multiple of three.

31. The standard Florida license plate is a combination of six alphanumeric characters. Some of the characters look similar to each other if you try to look at them from a distance (for example, try distinguishing a 'Q' from an 'O' at a distance). I was once told that they try to avoid issuing plates that have such similar characters. For example, they would never issue plates 'MAT ST8' and 'MAT 5T8' because the S and the 5 look the same from a distance. Based on this rule alone, how many possible license plates could Florida issue?

32. Let $f(x)=x^2+6x+1$, and let R denote the set of points (x,y) in the coordinate plane such that $f(x)+f(y) <= 0$ and $f(x)-f(y) <= 0$. What is the area of R?

33. I strongly believe in practicing for these competitions by going over old tests. So here is a question from an old test. What was the correct answer to question 33 of last year's State Convention Interschool test? (Hint: In the question, you had to deal with lots of anagrams)

34. The mean, median, unique mode, and range of a collection of 2004 integers are all equal to 2004. What is the largest integer that can be an element of this collection?

**Interschool Test**
**FAMAT State Convention 2004**

35. This is the story of a well-known man born years ago. He has influenced for many generations the thoughts and the minds of men and women in many different lands. The first and last digits of the year during which he was born add up to the second digit, the third digit is one larger than the second digit, and three times the fourth digit equals two times the third digit. Who is this man?

36. How many horses have fairly won horse racing's Triple Crown?

37. 2004 is a year for the Nth Summer Olympics, this year to be played in Athens, Greece. These Olympics started in 1896 also in Athens. What is the value of N?

38. A radio station has five one-hour talk shows, which listeners may call with questions. Each is on a different weekday (Monday-Friday), at a different time (between 5 PM and 10 PM), and on a different topic, with an emcee who uses a name appropriate to the topic (for example, the emcee for the show that takes personal questions uses the name "Dr. Friend"). From this information and the clues below, can you find out the day, time, and real full name of the emcee who hosts Dr. Plant's garden show?

1. Dr. Cash's finance show is earlier in the week and earlier in the day than Hall's; both are later in the day than Sue's show, but hers is between theirs in the week, though these shows are not necessarily on consecutive days.
2. Grant's show is broadcast the hour and the day after Dr. Plant's garden show.
3. No show is on in the same ordered place in the day and in the week; i.e., the Monday show is not on at 5 PM, the Tuesday one is not on at 6 PM, etc.
4. Sam, who hosts the Dr. Well health show, is on later in the day and earlier in the week than is Grant.
5. Liz's show is on one day later in the week and at a later hour than Conn's, and earlier in the week and one hour earlier in the day than Dr. Fixit's home repair show; of the other two, Ed's is earlier in the week and earlier in the day than Ford's.

39. The formula for the area of a regular octagon can be reduced to the form $(a+b \text{ root } c)x^2$, where x is the length of a side of the octagon, and a, b, and c are integers, $c>0$. What is the sum of a, b, and c? Assume that the function in parentheses is in its simplest form.

40. Suppose a goat is tethered to a corner of a building in the shape of a regular octagon, where the length of a side of the building is x, and the length of the tether is 2x. What is the area over which the goat can graze in terms of x?

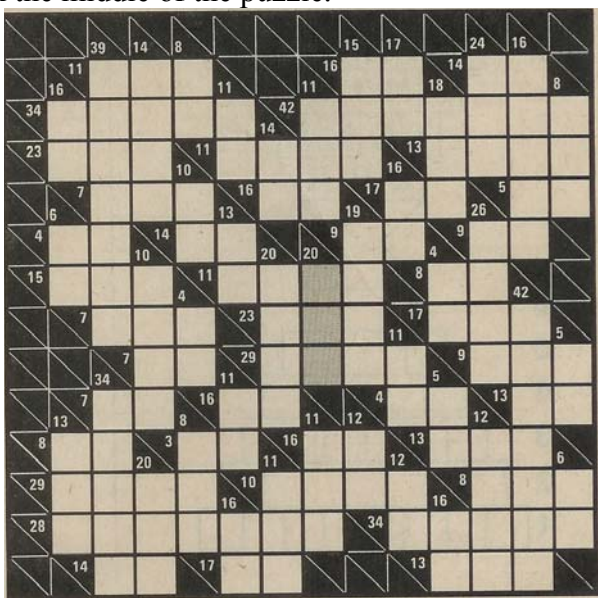
41. Forwarding e-mails is too easy of a way to clog up internet traffic. Suppose an e-mail forward that you receive asks you to send it on to ten people you know. If you do this, and everyone down the line continues to forward the message to ten other people, how many iterations, including the one you sent out, will it take to get to a billion addresses? Assume that the mail never goes to the same address twice.

42. In the addition problem below, the sum is given as well as the sum of the series of digits that appear in each line. Each dash is to be filled in with one of the digits 1 through 9. All digits will be used once and only once. On your answer sheet, right out the correct arithmetic for the problem.

```
    ___  ___  ___  ___  =  18
         ___  ___  ___  =  18
+             ___  ___  =  9
-------------------------------
      5    4    0    0
```

**Interschool Test**
**FAMAT State Convention 2004**

43. In this puzzle, the squares are filled in with numbers instead of letters. Each 'word' is a series of digits that add up to the white number in the black square. A number above a diagonal line indicates the digits are to be entered horizontally, to the right of the black square. A number below a diagonal line indicates the entry is a vertical one, to be entered below the black square. When solving the puzzle, keep in mind these rules: 1. Only the digits one through 9 are used, i.e., an entry cannot contain a 0; 2. An entry cannot contain a digit more than once; and 3. The digits in an entry may be in any order, but there is only one correct solution to the puzzle. On your answer sheet, I only want you to put in the number 'word' in the shaded region in the middle of the puzzle.



44. A pitcher throws a pitch in a standard major league baseball park that goes at a constant 100 mph. He releases the pitch at a point that is 6 feet closer to home plate than the pitcher's plate, and it travels in a straight horizontal line to the middle of the plate. How much time, to the nearest hundredth of a second, does it take for the pitch to travel from the release point to the rear point of home plate?

45. If an arc of $45^o$ on circle A has the same length as an arc of $30^o$ on circle B, then what is the ratio of the area of circle A to the area of circle B?

46. THIS QUESTION AND ITS ANSWER ARE WRITTEN ENTIRELY IN BASE 7. For a given arithmetic sequence $a_5=13$ and $a_{26}=130$. What is the value of $a_{21}$?

47. The college basketball season has just ended with what surely was an exciting final four (this test was written before all was finished!). Which college basketball program has the most wins in Division 1 history and by how many wins are they ahead?

48. Assuming the sky is clear on the first night of the convention (and assuming that night is April 16, 2004) at 9 PM, what planets of our solar system would be visible from here in Orlando (assuming you had a strong enough telescope to see all of the possible ones)?

49. What is the highest scoring word possible for a first turn in the game of Scrabble? How many points do you get from it?

50. How many terms are there in the expansion of $(a+b+c+d)^{2004}$?