

STATISTICS TEST – OPEN
FAMAT State Convention 2004

For all questions, E. NOTA means none of the above answers is correct.

1. A discrete random variable X takes four values and has the probability distribution shown below where a and b are constants. The expected value of X is 1.4. Find the value of $a^2 + b^2$.

X	0	1	2	3
$P(X)$	0.4	a	b	$a + b$

- A) 0.05 B) 0.29 C) 0.04 D) 0.10 E) NOTA
2. Given the facts below, find, $P[(A \cap \bar{B}) \cup (\bar{A} \cap B)]$.
 $P(A \cup B) = 0.6$, $P(A) = P(B)$, $P(B | A) = 0.8$
- A) 1/5 B) 1/4 C) 2/5 D) 1/3 E) NOTA
3. A student's GPA at Deerfield Beach High School is a continuous random variable that can lie anywhere on the interval $[0, 4]$. Robbie gathered the GPA of all the seniors at the school. He organized the data and found it interesting that the median was equidistant from the other two quartiles. He also found that if a senior's GPA was in the following interval $[0, 0.5) \cup (3.6, 4]$ then it could be considered an outlier. Find the value of the median. (Round to the nearest hundredth)
- A) 2.10 B) 2.05 C) 2.00 D) 2.15 E) NOTA
4. The least squares regression line of y on x was found to be $\hat{y} = mx + b$, where m and b are constants. The ratio of the coefficient of correlation to the coefficient of determination is 1.25 and $\frac{s_x}{s_y} = 2$. If $\bar{x} = 5$ and $\bar{y} = 4$, find the value of $m - b$.
- A) -1.6 B) 11 C) 5.6 D) -0.25 E) NOTA
5. Crystal found the $X\%$ confidence interval for the mean height in inches of 17 years old at her school to be (63,67), but she forgot the value of X . If she surveyed 20 students and the standard deviation of the height of 17 year olds at her school was known to be 6, help her find the value of X to the nearest percent.
- A) 86% B) 87% C) 88% D) 85% E) NOTA
6. A baseball player's slugging percentage is equal to the total number of bases a player gets divided by his number of at bats and his batting average is equal to his total number of hits divided by his number of at-bats. Players get 4 bases for a home run, 3 for a triple, 2 for a double, and 1 for a single. Chipper Jones lifetime batting average is .300. Home runs have accounted for 10% of his hits, triples for 20%, doubles of 30%, and singles for the rest. What is his lifetime slugging percentage? Express as a decimal to the nearest thousandth.
- A) 0.667 B) 0.750 C) 0.500 D) 0.600 E) NOTA

7. Let p be the probability that in 300 rolls of a pair of die the sum of the two die is a prime number between 100 and 140 times inclusive. Find the normal approximation (to the nearest hundredth) to the binomial for p with a continuity correction.
- A) 0.94 B) 0.92 C) 0.98 D) 0.96 E) NOTA
8. Let r equal the probability of getting two pair when you're playing poker with a standard 52-card deck. (Note that two pair means having two distinct pairs in a five-card hand.) Ken shuffles a 52-card deck and deals out a five-card hand. He repeats this process 199 more times and gets a two pair on a total of 4 hands. Ken believes the deck is not standard, so he runs the appropriate hypothesis test. His null hypothesis is that the probability of getting two pair on a hand from this deck is r , or in other words that this deck is standard. Find the p-value for the test to the nearest hundredth.
- A) 0.10 B) 0.09 C) 0.07 D) 0.08 E) NOTA
9. Dr. Brown and Dr. Benton are two neurosurgeons in Hyde Park. The following tables show the outcomes for their respective patients. In order for the data to show an example of Simpson's paradox (a reversal of comparison due to a confounding variable) a can take any integral value on the open interval (b, c) where b and c are integers. Find the value of $b - c$.

All Patients		
	Dr. Brown	Dr. Benton
Survived	190	200
Total	250	250

Patients in Good Condition		
	Dr. Brown	Dr. Benton
Survived	a	120
Total	68	137

Patients in Poor Condition		
	Dr. Brown	Dr. Benton
Survived	$190 - a$	80
Total	182	113

- A) 6 B) 11 C) 3 D) 4 E) NOTA
10. Six coins are tossed simultaneously 320 times. There were no tails 5 times, 1 tail 40 times, 2 tails 86 times, 3 tails 89 times, 4 tails 67 times, 5 tails 29 times, and 6 tails 4 times. Give the χ^2 statistic for the null hypothesis that all the coins are fair. (Round to the nearest hundredth.)
- A) 7.24 B) 7.86 C) 8.71 D) 10.92 E) NOTA
11. On what interval does the p-value for the test in problem 10 lie?
- A) $0.20 < p < 0.25$ B) $p > 0.25$ C) $0.10 < p < 0.20$ D) $p < 0.10$ E) NOTA

12. A researcher was testing the effectiveness of a cholesterol-increasing drug. The cholesterol of seven test subjects was measured before they started taking the drug and after a three-month period in which they were given the drug regularly. Perform the appropriate hypothesis test to figure out the effectiveness of the drug in increasing cholesterol. Find the p-value of the test. (Round to the nearest hundredth.)

Subject	A	B	C	D	E	F	G
Cholesterol Before	140	147	143	150	139	135	143
Cholesterol After	151	160	135	147	246	233	152

- A) 0.13 B) 0.09 C) 0.16 D) 0.11 E) NOTA
13. Andy Roddick's coach tells him that his first serve speed has a mean of 130mph and standard deviation of 6mph. Andy believes the mean is incorrect. He serves 10 times and finds his mean speed to be n , where n is a natural number. In order for the data to be significant at the 5% level, n cannot lie on the interval (c, d) but can lie on the interval $(0, c] \cup [d, \infty)$ where c and d are integers. Find $d - c$.
- A) 14 B) 7 C) 12 D) 6 E) NOTA
14. A random variable z is uniformly distributed between 0 and 20 inclusively. The probability that z is not within 0.5 units of a prime number can be written as $\frac{M}{N}$ where M and N are relatively prime integers. Find $M + N$.
- A) 31 B) 7 C) 8 D) 29 E) NOTA
15. A casino has a version of poker where you are dealt a five-card hand. It costs you \$1.00 to play, and you win nothing unless you get a full house (three of one kind and two of another). Let m be the amount (to the nearest cent) the casino would have to pay out for a full house if you want to break even in the long run. Find the ten-thousandths digit of $\log m$.
- A) 3 B) 1 C) 4 D) 9 E) NOTA
16. There are 30 marbles in a bag: 10 are green, 8 are blue, m are red, and n are yellow, where m and n are integers and $m \geq n$. (Note that 10, 8, m , and n add to 30.) When two marbles are selected from the bag without replacement the probability they are the same color is $107/435$. Find the value of $m^2 + nm + n$.
- A) 89 B) 100 C) 78 D) 111 E) NOTA
17. The ESPN / USA Today poll ranks the nation's top 25 college football teams. A computer program takes the rankings and randomly mixes them up. What is the probability that none of the teams are in their correct spots when compared to the real rankings? (Round to the nearest thousandth.)
- A) 0.363 B) 0.366 C) 0.369 D) 0.360 E) NOTA
18. The University of Chicago has an $n\%$ acceptance rate, and $m\%$ of those who are accepted decide to enter. Of the 9,000 applicants last year, 1,100 students made up the freshman class this year. Find the value of nm , to the nearest integer.
- A) 1222 B) 758 C) 1073 D) 1125 E) NOTA

19. The Cubs and the Marlins are playing a best-of-seven playoff series. Cubs' pitcher Mark Prior will start games 1 and 5. The Cubs have a 90% chance of winning a game pitched by Prior, and a 60% chance of winning every other game. What is the probability (to the nearest thousandth) that the Cubs win the series in 5 games?

- A) 0.467 B) 0.350 C) 0.389 D) 0.369 E) NOTA

20. Variables x and y are independent, with $\sigma_x = 2$, $\sigma_y = \sqrt{\tau}$, $\mu_x = \delta$, and $\mu_y = \tau$. If the $\sigma_{x+y} = \sqrt{10}$ and $\mu_{x+y} = \delta^2$ ($\delta > 0$), find the thousandths digit of $\ln(\tau + \delta)$.

- A) 1 B) 2 C) 6 D) 7 E) NOTA

21. In the previous problem, the relationship between the mean and the standard deviation of variable y is characteristic of the relationship between the mean and standard deviation of a variable that follows what type of distribution?

- A) Binomial B) Geometric C) Normal D) Multinomial E) NOTA

22. Let the function $f(x)$ be defined as follows, with b being a positive constant.

$$f(x) = \begin{cases} bx - 1 & c < x \leq c + 1 \\ b & c + 1 < x \leq c + 2 \\ -b(x - 3) + 1 & c + 2 < x \leq d \\ 0 & x \leq c \text{ \& } x > d \end{cases}$$

Let c and d be the x -intercepts for $y = bx - 1$ and $y = -b(x - 3) + 1$ respectively. For a certain value of b , $f(x)$ is a probability density curve. Using this value of b , find the hundredths digit of $\ln(b + c + d)$.

- A) 0 B) 2 C) 1 D) 7 E) NOTA

23. The mean and the standard deviation on the last test in Mr. Scales's history class were 75 and 8 respectively. Mr. Scales thought that the scores were too high. He is going to transform the scores so that the class mean becomes 65 and the standard deviation becomes 5. If Andrea got a 90 before the transformation, what score will she end up with after the "curve" is applied? (Round to the tenths place.)

- A) 66.3 B) 80.3 C) 74.4 D) 78.0 E) NOTA

24. The power for a hypothesis test run at a 5% significance level was 0.86. Find the sum of the probability of a Type I error and the probability of a Type II error.

- A) 0.14 B) 0.09 C) 0.19 D) 0.05 E) NOTA

25. A boat has seats for four men and a weight limit of 800 pounds. The distribution of the weight of adult males is normal with a mean of 190 pounds and a standard deviation of 10 pounds. If the weights of individual passengers are independent, what is the probability that a group of 4 males exceeds the boat's weight limit? (Round to the nearest thousandth.)

- A) 0.023 B) 0.159 C) 0.001 D) 0.067 E) NOTA

26. If one of the distinguishable combinations of the word MISSISSIPPI is chosen at random, what is the probability that it starts and ends with a P? (Round to the nearest thousandth.)
- A) 0.018 B) 0.011 C) 0.009 D) 0.021 E) NOTA
27. An increase in which of the following values (while everything else remains constant) will unambiguously increase the probability of committing a Type II error when running a one sample z-test?
- A) Sample size
B) Population size
C) Population variance
D) Significance level
E) NOTA
28. Out of 100 students in a school, 20 do not take analysis or linear algebra and 20 take both analysis and linear algebra. The total number of students that take analysis is equal to the total number of students that take linear algebra. How many students take linear algebra?
- A) 60 B) 30 C) 50 D) 20 E) NOTA
29. During the New Hampshire primary, exit polls were conducted in the following manner at one polling place. Every third person that walked out of the voting booth was handed a survey asking how he or she had voted. This is an example of what type of sampling?
- A) Simple Random Sampling
B) Stratified Sampling
C) Proportional Sampling
D) Systematic Sampling
E) NOTA
30. The least squares regression line of y on x was calculated to be $\hat{y} = ax + b$, where a and b are constants, and $\bar{x} = 15$ and $\bar{y} = 26$. One of the data points used to make the line was $(x, y) = (3, 6)$. If this point has a residual of 4, find the thousandths digit of $\sqrt[3]{a}$.
- A) 6 B) 7 C) 1 D) 2 E) NOTA