

LOGS AND EXPONENTS: THETA

FAMAT State Convention 2004

For all questions, E. NOTA means none of the above answers is correct.

- 1) Solve for a :
 $\log_7(a - 4) = 2$
 A) 18 B) 49 C) 53 D) 104 E) NOTA
- 2) Solve for b :
 $2 \cdot 4^{b-2} = 32$
 A) 2 B) 4 C) 8 D) 16 E) NOTA
- 3) What is the unit's digit of $2^{2004} + 3^{2004} - 5^{2004}$?
 A) 0 B) 2 C) 5 D) 8 E) NOTA
- 4) Which of the following is the same representation of the function
 $h(x) = 16^{\sqrt{x}}$, $x > 0$?
 I. $h(x) = 4^x$ II. $h(x) = 8 \cdot 2^{\sqrt{x}}$ III. $h(x) = 8^{\sqrt{x}}$ IV. $h(x) = 4^{2\sqrt{x}}$
 A) IV. only B) I. & IV. only C) II. & III. only D) All of these
 E) NOTA
- 5) Find the sum of all values of a that satisfy the given equation:
 $\left| \begin{array}{cc} \log_{10} a & -1 \\ \log_{10}(a-1) & 2 \end{array} \right| = \log_{10} a - \log_{10}\left(\frac{1}{2}\right)$, $a > 0$
 A) 0 B) 1 C) 2 D) No solution E) NOTA
- 6) Given: $9^x - 2 \cdot 3^{x+1} - 7 = 0$
 The solution for x to the equation above can be represented as a decimal. What is the hundredths digit of the decimal representation?
 A) 0 B) 1 C) 6 D) 7 E) NOTA
- 7) Evaluate:
 $\sqrt{182 + \sqrt{182 + \sqrt{182 + \dots}}}$
 A) 13 B) $\sqrt{182}$ C) 14 D) ∞ E) NOTA

- 8) The number of bacteria in a certain sample is monitored by a scientist. He notes that after one week there are 6531 bacteria and after two weeks there are 8634. If the amount of bacteria present follows a logarithmic pattern ($y = a + b \ln x$) where y is the amount of bacteria after x weeks, how many whole bacteria are in the sample after three weeks?
 A) 9864 B) 10351 C) 10734 D) 13769 E) NOTA
- 9) Evaluate: $\sum_{n=1}^4 \log_2 n$
 A) 0 B) 1 C) 2 D) $\log_2 10$ E) NOTA
- 10) If $a = \log_{10} x$, $b = \log_{10} y$, and $c = \log_{10} z$, where x , y , and z are all greater than 0, then which of the following is equal to $2a - b + \frac{3c}{2}$?
 A) $\log_{10} \left(\frac{3 \cdot x \cdot z}{y} \right)$ B) $2 \log_{10} \left(\frac{x \cdot \sqrt[4]{z^3}}{\sqrt{y}} \right)$ C) $\log_{10} \left(\frac{x^2 \cdot \sqrt[3]{y^2}}{z} \right)$
 D) $3 \log_{10} \left(\frac{x \cdot z}{y} \right)$ E) NOTA
- 11) Simplify: $2^{\log_4 2^{100}}$
 A) 2^{25} B) 2^{50} C) 2^{100} D) 2^{200} E) NOTA
- 12) Given: $\log_{10} A = \frac{5}{2}$ $\log_{10} B = \frac{9}{2}$ $\log_{10} C = -\frac{3}{2}$
 What is the value of $\log_{10} \left(\frac{A \cdot B^2}{C^3} \right)$?
 A) -15 B) 7 C) 15 D) 16 E) NOTA
- 13) The graphs of $y = x^2$ and $y = 2^x$ intersect in how many places?
 A) 0 B) 1 C) 2 D) 3 E) NOTA
- 14) Which of the following function(s) has or have inverses that are functions for a domain of all real numbers?
 I. $y = x^2 - 1$ II. $2y = x + 3$ III. $y = x^3 - 6$ IV. $y = x^3 - 6x$
 A) II. only B) I. & IV. only C) II. & III. only D) All of these
 E) NOTA

- 15) Simplify the given expression in terms of x : $\left(-1024^{\frac{1}{2}}x^2\right)^{\frac{-4}{5}}$, $x \neq 0$
- A) $\frac{1}{16x^{8/5}}$ B) $\frac{1}{32x^2}$ C) $-\frac{1}{4x^{-6/5}}$ D) Undefined E) NOTA
- 16) Given: $f(x) = \log_2(x!+8)$, $x \in \mathbb{N}$
Find: $f^{-1}(5)$
- A) 4 B) 5 C) 7 D) Cannot be determined
E) NOTA
- 17) The BANK OF WYΩING compounds its money continuously according to the formula: $y = Pe^{rt}$. Ms. Fish decides to put \$1,525.00 in this bank and after 1 year, she discovers that she now has \$1,642.13. How long will it take for her money to double in value? (Round your answer to the nearest tenth of a year)
- A) 1.9 B) 6.7 C) 9.4 D) 13.2 E) NOTA
- 18) The number q written in scientific notation is given by: $q = 6.63 \times 10^3$.
In this form let A be the characteristic of q and B be the mantissa of q . Using this information, solve for x in the given equation: (Round your answer to the nearest hundredth)
- $$4^{(Ax)} = 3^{(Bx+1)}$$
- A) -0.35 B) 0.19 C) 2.41 D) 2.82 E) NOTA
- 19) What is the coefficient of the $a^{12}b^{12}$ term of the given expansion?
- $$\left(\frac{2}{3}a^3 + \frac{1}{2}b^2\right)^{10}$$
- A) $\frac{1}{324}$ B) $\frac{5}{18}$ C) $\frac{35}{54}$ D) $\frac{28}{27}$ E) NOTA
- 20) What is the domain of $y = \log_4(x^2 - 2)$?
- A) $x \in \mathbb{R}$ B) $x > 0$ C) $x < -2$ or $x > 2$ D) $x \leq -\sqrt{2}$ or $x \geq \sqrt{2}$
E) NOTA

- 21) Find an expression in terms of n for: $\prod_{x=3}^n \log_x(x+1)$ $n \in N$
 A) $\log_n(n+1)$ B) $\log_3 n$
 C) $\log_3(n+1)$ D) $\log_n 4$ E) NOTA
- 22) Using the fact that $i = \sqrt{-1}$, simplify the given expression:

$$\left[(-i)^{-1} + \left(\frac{1}{i^3} \right)^5 \right]^4$$

 A) -4 B) 0 C) 1 D) 16 E) NOTA
- 23) Given: $\log_{x^2} y = \frac{1}{9}$ $x > 0, y > 0$
 Find: $\log_{\sqrt{y}} x^3$
 A) 9 B) $\frac{27}{2}$ C) 18 D) 27 E) NOTA
- 24) Given: $50^a 125^b 8^c = 1000$
 Find: $a + b + c$
 A) 2 B) 3 C) 4 D) 6 E) NOTA
- 25) Find the sum of all values of y that satisfy the given equation:
 $\log_y(y+12) = 2$, $y > 0$
 A) 0 B) 1 C) 4 D) 12 E) NOTA
- 26) Evaluate: $\sum_{z=1}^{\infty} \frac{1}{2^z}$
 A) $\frac{1}{2}$ B) 1 C) 2 D) $\frac{7}{4}$ E) NOTA
- 27) Solve for x : $\log_7(\log_5(\log_2 x)) = 0$
 A) 2 B) 4 C) 16 D) 32 E) NOTA

- 28) Given: $x = a^r$, $y = a^{2r}$, and $z = 2x$ for a , x , y , and z all greater than 1. How many of the following are true?

I. $\log_x a = r$ II. $y = x^2$ III. $2y = z^2$ IV. $\log_a \left(\frac{z}{2} \right) = r$

A) 1 B) 2 C) 3 D) 4 E) NOTA

- 29) Given:
$$a \# b = \begin{cases} a^b & \text{if } a < b \\ \left(\frac{a-5}{3} \right) \# (b+1) & \text{if } a \geq b \end{cases}$$

What is the value of $146 \# 1$?

- A) 16 B) 27 C) 81 D) 146 E) NOTA
- 30) How many digits are in 2004^{2004} ?
- A) 6617 B) 6618 C) 8016 D) 8017 E) NOTA