1. Let $f\left(x\right)=\dfrac{x^2+2x-3}{x+7}$. Find $f''\left(-3\right)$.

A. -1 B. $\dfrac{3}{8}$ C. 1 D. $\dfrac{81}{64}$ E. NOTA

2. If $y=3^{\log_2 x}$, find $y'\left(2\right)$.

A. 1 B. $\log_4 3$ C. $\log_4 27$ D. 3 E. NOTA

3. Evaluate: $\displaystyle\int_1^{\sqrt{e}} x\ln x\,dx$

A. $\dfrac{1}{4}$ B. $\dfrac{1}{2}$ C. $\dfrac{e}{4}$ D. $\dfrac{e}{2}$ E. NOTA

4. Let $f\left(x\right)=\displaystyle\int_{-x}^{x}\cos\left(a\right)da$ be defined for $x\geq 0$. If *A* is the maximum value of *f* and if *B* is the minimum value of *f*, then what is the value of $A-B$?

A. $-\pi$ B. 0 C. 1 D. 2 E. NOTA

5. Evaluate: $\displaystyle\sum_{i=0}^{2006}\binom{2006}{i}\cdot(-2)^i$

A. -1 B. 0 C. 1 D. 2006 E. NOTA

6. If $f\left(x\right)=\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\cdots}}}}$, find $f'\left(2\right)$.

A. ¼ B. ⅓ C. ½ D. 2 E. NOTA

7. Evaluate $\displaystyle\int_1^2\frac{1}{\left[x^2\right]}dx$, where $\left[x\right]$ denotes the largest integer less than or equal to *x*.

A. $\dfrac{1}{2}$ B. $\dfrac{\sqrt{3}}{6}+\dfrac{\sqrt{2}}{2}-\dfrac{1}{3}$ C. $\dfrac{1+\sqrt{2}+\sqrt{3}}{6}$ D. 1 E. NOTA

8. Let f and g be differentiable functions such that f is even and g is odd. If $f(5)=6$, $f'(5)=2$, $g(5)=-5$, and $g'(5)=-1$, find $h'(-5)$ where $h(x)=\dfrac{f(x)}{\left[g(x)\right]^2}$.

A. $-\dfrac{4}{25}$ B. $-\dfrac{2}{125}$ C. $\dfrac{2}{125}$ D. $\dfrac{4}{25}$ E. NOTA

9. Evaluate: $\displaystyle\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

A. $\dfrac{1}{2}e^2$ B. $2e(e-1)$ C. $2e^2-2$ D. $4e^2$ E. NOTA

10. Jen is the owner of a perfectly competitive widget manufacturer. Being a member of a perfectly competitive market, Jen has no input on the price of widgets. The market dictates that a case of widgets sells for \$100 each. The cost of producing x cases of widgets is given by the function $C(x)=.05x^2-10x$. Additionally, Jen has a fixed cost of \$1000 to pay the rent on the building where she produces her widgets. If Jen wants to maximize her profit, how many cases should she produce? Assume that Jen sells all of the widgets that she produces.

A. 0 B. 100 C. 500 D. 1100 E. NOTA

11. Let x and y be uniformly distributed, independent random variables such that $x\in[5,10]$ and $y\in[0,10]$. What is the probability that $x+y>xy$?

A. $\dfrac{1}{2}e^{-\frac{3}{2}}$ B. $\dfrac{1}{10}+\dfrac{1}{25}\ln\dfrac{3}{2}$ C. $\dfrac{1}{2}-\dfrac{1}{2}\ln 2$ D. $\dfrac{2}{11}$ E. NOTA

12. If $\displaystyle\lim_{x\to 0}\frac{axe^x}{\sin(bx)}=1$, then what is the value of $a-b$?

A. 1 B. 2 C. e D. Cannot be determined E. NOTA

13. Arrange the values of the following definite integrals in increasing order.

I. $\displaystyle\int_2^5 \ln\left(x^2+1\right)dx$ II. $\displaystyle\int_2^5 x\cdot 2^x\,dx$ III. $\displaystyle\int_2^5 x\cdot\sqrt[3]{x^4+1}\,dx$ IV. $\displaystyle\int_2^5 \frac{1}{x^4+2x^2+1}\,dx$

A. I, IV, III, II B. II, IV, I, III C. IV, I, II, III D. IV, I, III, II E. NOTA

14. Evaluate: $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i}{i^2+n^2}$

A. $\ln\sqrt{2}$ B. $\ln 2$ C. 1 D. e E. NOTA

15. What is the maximum value of n such that $2006!$ is divisible by 2^n?

A. 500 B. 1003 C. 1998 D. 2047 E. NOTA

16. Evaluate: $\int_{-\pi}^{\pi}\left|\sin(x)\right|dx$

A. 0 B. 2 C. π D. 4 E. NOTA

17. Given $\int_{-\infty}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}dx=1$ for constants μ and σ where $\mu\in\mathbb{R}$ and $\sigma>0$, evaluate

$\int_{-\infty}^{\infty}\frac{1}{\sqrt{\pi}}e^{-\frac{x^2}{3}}dx$.

A. $\frac{\sqrt{6}}{2}$ B. $\sqrt{3}$ C. $\sqrt{6}$ D. $\sqrt{3\pi}$ E. NOTA

18. Evaluate: $\lim_{x\to 0}\left(\frac{1}{x\cos(x)}\int_{x^2}^{x}e^{t^2}\,dt\right)$

A. 0 B. 1 C. e
D. The limit does not converge E. NOTA

19. What is that smallest integer n such that $2.01^n-2^n>10$?

A. 6 B. 8 C. 10 D. 12 E. NOTA

For questions 20 and 21, use the following information:

Alex and Brian are kicking soccer balls in a large, flat field. Alex decides to challenge Brian to see who can kick the ball the furthest. Brian kicks the ball with an initial velocity of 64 ft/sec. Brian kicks the ball at an angle of inclination of 30 degrees and Alex kicks the ball at an angle of inclination of 45 degrees. Brian and Alex kick the ball at the same time. Assume that acceleration due to gravity is 32 ft/sec. Also assume that there is no wind and ignore air resistance.

20. If the balls collide in midair, what is the initial velocity of Alex's ball? Express your answer in ft/sec.

A. $16\sqrt{2}$ B. 32 C. $32\sqrt{2}$ D. $32\sqrt{3}$ E. NOTA

21. If the balls do not collide and land at the same time, then who would kick the ball the furthest?

A. Alex B. Brian C. Both kick the same distance
D. Not enough information E. NOTA

22. The region bounded by the graphs $y = x^2$ and $x = y^2$ is rotated about the line $x = -3$. Find the area of the resulting solid.

A. $\frac{3\pi}{10}$ B. $\frac{13\pi}{10}$ C. $\frac{23\pi}{10}$ D. $\frac{33\pi}{10}$ E. NOTA

23. Evaluate: $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}^{2006}$

A. $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ B. $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ C. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ E. NOTA

24. Let $\frac{dy}{dx} = \frac{x}{y^2}$. If $y(4) = 3$, find $y(0)$.

A. 0 B. 1 C. $\sqrt[3]{3}$ D. $3 \cdot \sqrt[3]{2}$ E. NOTA

25. Let X be a continuous random variable with probability density function $f(x) = \begin{cases} Cxe^{-x^2} & x > 0 \\ 0 & \text{otherwise} \end{cases}$.

What is the value of C?

A. $\frac{1}{\sqrt{2\pi}}$ B. $\frac{1}{2}$ C. 1 D. 2 E. NOTA

26. Suppose two goods *X* and *Y* are the only goods being sold in a market. An indifference curve is a curve showing a series of bundles of goods *X* and *Y* that yield the same utility to a consumer, hence the consumer is indifferent between all consumption bundles on the indifference curve. A consumer's indifference map is the set of all possible indifference curves for a particular consumer. Suppose Lauren's indifference map is given by $U(x,y)=x^2y$ where $U(x,y)$ is the utility associated with consumption of *x* units of good *X* and *y* units of good *Y*. Suppose good *X* costs \$2 per unit and good *Y* costs \$4 per unit. Lauren has \$300 to spend on goods *X* and *Y*. How many units of *X* will Lauren buy if she wishes to maximize her utility? Assume that only whole number units of goods *X* and *Y* can be purchased.

A. 10　　B. 25　　C. 100　　D. 150　　E. NOTA

27. Determine the number of vectors $(x_1,\ldots,x_9)$ such that each x_i is either 0 or 1 and $\sum_{i=1}^{9} x_i \geq 5$.

A. 126　　B. 256　　C. 512　　D. 362880　　E. NOTA

28. Evaluate the derivative of $x^2 \ln x$ with respect to $\frac{1}{x}$, where $x > 0$.

A. $-\frac{1}{x^3}\left(2\ln\frac{1}{x}+1\right)$　　B. $\frac{1}{x}\left(2\ln\frac{1}{x}+1\right)$

C. $x(2\ln x+1)$　　D. $-x^3(2\ln x+1)$　　E. NOTA

29. Evaluate: $\lim_{x\to\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2+2x}\right)$

A. 0　　B. $\sqrt{3}-\sqrt{2}$　　C. $\frac{3}{2}$　　D. The limit does not converge　　E. NOTA

30. The base of a solid is the circular region in the *xy*-plane bounded by the graph of $x^2+y^2=a^2$, where $a>0$. Every cross section by a plane perpendicular to the *x*-axis is a square. If the volume of the solid is 144, what is the value of *a*?

A. 3　　B. $3\cdot\sqrt[3]{2}$　　C. $3\cdot\sqrt[3]{3}$　　D. $3\cdot\sqrt[3]{4}$　　E. NOTA