1. A
2. A
3. A
4. D
5. A
6. C
7. A
8. B
9. A
10. E
11. A
12. A
13. B
14. A
15. C
16. A
17. E
18. D
19. B
20. D
21. E
22. D
23. C
24. B
25. B

1. **A**

2. **A** Fermat's Last Theorem said that for integers a, b, c, and n ($n > 2$), the solution $a^n + b^n = c^n$ has no solutions. **A** is of this form. **B** is of this form but for $n = 2$, so we can't disprove it using FLT. **C** doesn't apply to FLT because the exponents are different, and **D** has too many terms on the left.

3. **A** $\left(1^4+2^4+3^4+\ldots+n^4+(n+1)^4\right)-\left(1^4+2^4+3^4+\ldots+n^4\right)=(n+1)^4$

.

4. **C** $f(n+1)-f(n)=g(n+1)-g(n) \Rightarrow f(n)-f(n-1)=g(n)-g(n-1)$. What we've been told is that f and g have the same difference between every function value. If $f(n) = g(n)$ for some n, then plugging into the above two equations gives $f(n + 1) = g(n + 1)$ and $f(n - 1) = g(n - 1)$. Repeating this process for $n + 1$ and $n - 1$ we can inductively show that they are equal at all values of n. Thus any point where they're equal implies equality at all points. However it is important to notice that the sum definition of $f(n)$ is undefined for values of n less than 1 (you can't sum from 1 to something less than 1). For this reason, **I** is no good because $f(0)$ is undefined.

5. **A** Since $\sum_{i=1}^{n} i$ is a 2nd-degree polynomial, $\sum_{i=1}^{n} i^2$ is 3rd-degree, etc., it's safe to say we're looking for a 5th-degree polynomial. Since 5th-degree polynomials can be uniquely determined by 6 points, you have only to plug in up to 6 values to be sure **A** always works and the others don't.

6. **C** This is a by-the-book inductive proof; first prove that if something is true for n, it's also true for $n + 1$, then find some n where it's true and you're done!

7. **A** 1 is the only positive integer that can divide n and $n + 1$ for some n. Every prime will divide the product of all primes, so (since 1's not prime) none of them can divide 1 more than this product.

8. **B** This proof is indirect because it first assumes the opposite of what's being proven, and shows that this leads to a contradiction.

9. **A** A corollary is a result or special case. We can obtain Heron's from Brahmagupta's by imagining a quadrilateral with one side length 0 (a triangle). If you plug $d = 0$ into Brahmagupta, you get Heron. This "quadrilateral" also necessarily has a circumscribed circle because all triangles do.

10. **D** The analogy is a theorem to its corollary. Rolle's Theorem is just a flat version of Mean Value Theorem, MacLauren series are just 0-centered versions of Taylor series, Pythagorean Theorem is just a right-angle version of Law of Cosines, and Law of Sines is just a reciprocated version of the Extended Law of Sines, minus the part about R (Extended Law of Sines:

$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$, where R is the circumradius of the triangle).

11. **A**

$$(ABC) = (BCD) + (ACD) + (ABD) = \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr = \frac{1}{2}(a+b+c)r = \frac{1}{2}rP = A \Rightarrow 2A = rP$$

12. **A** For a tetrahedron ABCD with incenter E, (ABCD) = (ABCE) + (ABED) + (AECD) + (EBCD). Since the volume of any pyramidal solid is one third the base area times the altitude to that base, *(ABCE) = ⅓(*ABC*)r* , *r* being the inradius as well as the altitude from E to faces ABC, ABD, ACD, and BCD.

So $$(ABCD) = \frac{1}{3}r\big((ABC) + (ABD) + (ACD) + (BCD)\big) = \frac{1}{3}Ar = V \Rightarrow rA = 3V$$

13. **B: I** can be proved simply by noting that going around any two sides of a triangle is obviously a longer path than if you had just traveled directly between them. **II** and **III** are much more complex, and probably wouldn't utilize Euclid's statement anyways. **IV** is just another way of saying "going around the edge of a circle takes longer than cutting straight across," i.e. half the circumference is longer than the diameter, so the whole circumference is longer than twice the diameter. This uses nothing but the given statement and some arithmetic.

14. **A** This is a basic proof of the Pythagorean Theorem, $a^2 + b^2 = c^2$.

15. **C** Just look at the diagram...

16. **A** (The proof is valid.)

17. **E** (The proof is valid.)

18. **D** Anytime $b = 1$ and a is one more than a prime, $a^b - 1$ is prime. For example $8^1 - 1 = 7 =$ prime.

19. **B** When $a = 2$, the stated factors 1 and $a - 1$ are equal, and there are only 2 guaranteed factors, leaving the possibility that $a^b - 1$ will be prime. $2^2 - 1$, $2^3 - 1$, $2^5 - 1$, among many others (see Mersenne Primes) are prime. However, for $a > 2$ and $b > 1$, the three stated factors are distinct, and $a^b - 1$ is never prime.

20. **D** See #18, #19. It is possible for 1, $a - 1$, and $a^b - 1$ to not be distinct, in which case it is possible for $a^b - 1$ to have only 1 or 2 factors.

21. **D** First notice that any number x can be shown to equal any other number y using the erroneous conclusion that 2 = 1:

$2 = 1$	"Proven"
$1 = 0$	Subtracting 1 from both sides
$x - y = 0$	Multiplying by $x - y$ on both sides
$x = y$	Adding y to both sides

So, since we can show that 2 = 0, and since Winston Churchill had 2 arms and 2 legs, this means he had 0 arms and 0 legs. And since 0 = 1, and Winston had 0 green, leafy tops, he must have had 1 green, leafy top. Let his waist size (in centimeters) be W. Since $W = 0$, he tapered to a point at the waist. Lastly, let the wavelength of orange light be *Orange* and let the wavelength of light that reflected off Winston's skin be *Winston*, and we can show that *Winston* = *Orange*! So, to recap, he had no arms or legs, tapered to a point, was orange, and had a green, leafy top. Clearly, he was a carrot! (Don't dispute this or I'll prove that you're something far worse!)
Alternately, starting with 1 = 0 as above, we can multiply both sides by (*Winston Churchill*) – (*Carrot*), then add (*Carrot*) to both sides, yielding:

Winston Churchill = *Carrot*

This may better satisfy the less imaginative, more left-brained test-takers. (The gist of this "proof" was paraphrased from <u>Zero: The Biography of a Dangerous Idea</u> by Charles Seife.)

22. **D** In the step where I divide by $b - a$, since the original condition was that $a = b$, $b - a = 0$ and from here on the statements become false because I have divided by 0. Basically what has happened is that since 2(0) = 1(0), I said 2 = 1 by canceling the 0's!

23. **C**

24. **B** In order the statements are worth 1, 2, 2, 1, 1, and 1 points. Play with them if you don't believe me.

25. **B**