Throughout this test, let ($V_1V_2\ldots V_n$) denote the area of the polygon with vertices V_1, V_2, …, Vn, and let a|b mean "a divides b," or a is a factor of b.
The divider lines indicate the start/end of a series of related questions.

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1. A monumental pursuit involving millions of man-hours, false proofs, and over 200 years was started when a now-famous mathematician scribbled the following note in the margin of a book: "I have found a truly marvelous proof of this which this margin is too narrow to contain." What did the theorem he was offering to prove come to be known as?

A) Fermat's Last Theorem B) The Riemann Hypothesis
C) The Pythagorean Theorem D) The Four Color Theorem E) NOTA

2. The theorem was finally proved in 1993. Which of the following statements can be proven false directly as a result of it?

A) $314159265358979^{10} + 271828182845904^{10} = 320866050087045^{10}$
B) $10^2 + 15^2 = 18^2$ C) $2^3 + 4^5 = 6^7$ D) $3^4 + 4^4 + 5^4 = 6^4$ E) NOTA

Everyone knows $1 + 2 + 3 + \ldots + n = \dfrac{n(n+1)}{2}$, $1^2 + 2^2 + 3^2 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$, and

$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\dfrac{n(n+1)}{2}\right)^2$.

Suppose $f(n)$ is the function such that for any non-negative integer n, $0^4 + 1^4 + 2^4 + 3^4 + \ldots + n^4 = f(n)$.

3. What is $f(n+1) - f(n)$?

A) $(n+1)^4$ B) n^4 C) $(n-1)^4$ D) $(n+1)^4 - n^4$ E) NOTA

4. Suppose some function g satisfies the condition that $g(n+1) - g(n) = f(n+1) - f(n)$ for all non-negative integers n. Which of the following additional pieces of information would guarantee that $g(n) = f(n)$ for all non-negative integers n?

I. $g(0) = 0$ **II.** $g(1) = 1$ **III.** $g(2) = 17$

A) **I** only B) **II** only C) **II** and **III** only D) **I**, **II**, and **III** E) NOTA

5. Which of the following is $f(n)$?

A) $\dfrac{n^5}{5} + \dfrac{n^4}{2} + \dfrac{n^3}{3} - \dfrac{n}{30}$ B) $\dfrac{6n^5 + 15n^4 + 10n^3 - n + 30}{30}$

C) $\dfrac{5n^4}{2} - \dfrac{20n^3}{3} + 10n^2 - \dfrac{29n}{6}$ D) $\dfrac{5n^5}{24} + \dfrac{5n^4}{12} + \dfrac{5n^3}{8} - \dfrac{5n^2}{12} + \dfrac{1}{6}$ E) NOTA

6. One could prove that the previous answer is indeed our desired $f(n)$ by noting that they have the same difference between function values (Question 4). Then, if they are equal at any point, they must be equal at every point to the right (and left) of that. Officially, this type of proof/reasoning is described as:

A) Direct B) Indirect C) Inductive D) Deductive E) NOTA

7. Over 2000 years ago, Euclid proved that there must be an infinite number of prime numbers. His argument went something like this: "Suppose there are only a finite number of primes, and we label them p_1, p_2, p_3, …, p_n, n being some finite number. Then **X** must be a prime or a product of primes that is(are) not yet part of our list." What is **X**?

A) $1+\prod_{i=1}^{n} p_i$ B) $2+\prod_{i=1}^{n} p_i$ C) $3+\prod_{i=1}^{n} p_i$ D) $5+\prod_{i=1}^{n} p_i$ E) NOTA

8. What is this kind of proof known as?

A) Direct B) Indirect C) Inductive D) Deductive E) NOTA

Heron noted that the area of a triangle with side lengths a, b, and c is given by:

$$\frac{\sqrt{(a+b+c)(a+b-c)(a-b+c)(-a+b+c)}}{4}$$

Later, Brahmagupta noted that the area of a cyclic quadrilateral with side lengths a, b, c, and d is given by

$$\frac{\sqrt{(a+b+c-d)(a+b-c+d)(a-b+c+d)(-a+b+c+d)}}{4}$$

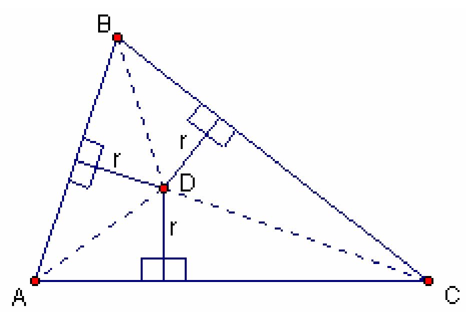
9. What is the relationship between these two theorems?

A) Heron's is a corollary of Brahmagupta's B) Brahmagupta's is a corollary of Heron's
C) Both can be derived from Robbins' Theorem D) No relationship E) NOTA

10. Brahmagupta's Theorem : Heron's Theorem =

I. Mean Value Theorem for Derivatives : Rolle's Theorem **II**. Taylor Series : Maclauren Series
III. Law of Cosines : Pythagorean Theorem **IV**. Law of Sines : Extended Law of Sines

A) **II** and **IV** only B) **I** and **III** only C) **I**, **III**, and **IV** only D) **I**, **II**, **III**, and **IV** E) NOTA



11. Consider the following diagram at left and the statement below: (ABD) + (ACD) + (BCD) = (ABC).
Which of the following equations is true for every triangle with inradius *r*, perimeter *P*, and area *A*, and is proved using the given statement and diagram?

A) $rP = 2A$ B) $rP = A$
C) $2rP = A$ D) $r^2 = 2AP$ E) NOTA

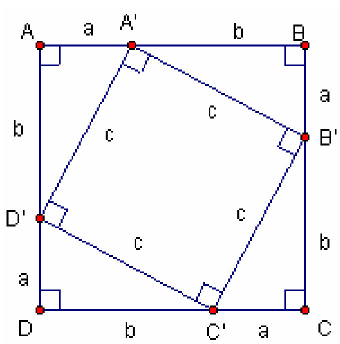
12. Generalizing the previous equation to a tetrahedron with surface area *A*, volume *V*, inradius *r*, and perimeter (sum of edge lengths) *P*, we get:

A) $rA = 3V$ B) $PA^2 = 2Vr^2$ C) $r^2A = PV$ D) $rA = V$ E) NOTA

13. Euler said "the shortest distance between any two points is a straight line." This by itself can be used to prove which of the following statements?

I. If *a*, *b*, and *c* are the lengths of the sides of a triangle, then $a + b > c$, $a + c > b$, and $b + c > a$.
II. The intersection of a triangle's medians is its center of mass.
III. The sum of the exterior angles of any convex polygon is 360º.
IV. The circumference of a circle is more than twice the diameter.

A) **I** only B) **I** and **IV** only C) **II** and **IV** only D) **I**, **III**, and **IV** only E) NOTA

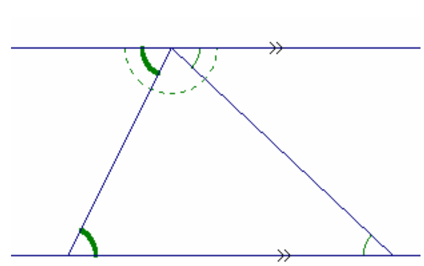


14. The diagram at left and the statement below are key to proving what famous equation?

(ABCD) = (A'B'C'D') + (AA'D) + (BB'A) + (CC'B) + (DD'C)

A) Pythagorean Theorem B) Triangle Area Formula
C) Law of Cosines D) Triangle Inequality E) NOTA

15. Knowing properties of parallel lines, transversals, and alternate interior angles, which fundamental principle of geometry is illustrated by the following diagram?



A) The sum of a polygon's exterior angles is 360º.
B) The sum of any 2 sides of a triangle is greater than the third.
C) The sum of a triangle's interior angles is 180º.
D) A triangle's area is half the product of the base and height.
E) NOTA

Consider the following "proof" of the statement (which may or may not be true):
"The equation $a^4 = b^2 + 11$ has no solutions for integers a, b."

Proof: $a^4 = b^2 + 11 \Rightarrow (a^2 - b)(a^2 + b) = 11$. Since integers are closed over addition, subtraction, and multiplication, if a and b are integers, $a^2 - b$ and $a^2 + b$ must also be integers. The only pairs of integers that multiply to 11 are (11, 1) and (-11, -1). Assuming b is positive, $a^2 - b$ must be the lesser factor in either case, and neither of the systems of equations $\left\{\begin{matrix} a^2 - b = 1 \\ a^2 + b = 11 \end{matrix}\right\}$ or $\left\{\begin{matrix} a^2 - b = -11 \\ a^2 + b = -1 \end{matrix}\right\}$ have solutions in integers a and b, so the initial statement is true!

16. In fact, how many ordered pairs of integers (a, b) satisfy the equation $a^4 = b^2 + 11$?
A) 0 B) 1 C) 2
D) More than 2, but a finite number E) NOTA

17. What (if there is one) is the flaw in the "proof?"
A) A special case is neglected. B) The systems are solvable. C) There is an algebraic error.
D) Permutations of the factorizations of 11 are not considered. E) NOTA

Consider the following "proof" of the statement (which may or may not be true):

"There do not exist positive integers a, b such that $a^b - 1$ is prime."

Claim:	***Justification:***
$a \equiv 1 \bmod (a-1)$	Modular Arithmetic
$\Rightarrow a^b - 1 \equiv 1^b - 1 \equiv 1 - 1 \equiv 0 \bmod (a-1)$	Modular Arithmetic
$\Rightarrow (a-1) \mid (a^b - 1)$	Modular Arithmetic
$1 \mid (a^b - 1)$	Trivial; 1 divides all integers
$(a^b - 1) \mid (a^b - 1)$	Trivial; every integer divides itself
$\Rightarrow a^b - 1$ is not prime!	$a^b - 1$ has three definite factors: $1,\ a-1,\ a^b - 1$. Primes must have exactly 2 factors!

18. How many distinct positive integers a exist such that, for some positive integer b, $a^b - 1$ is prime?
A) 0 B) 1 C) 2 D) Infinitely many E) NOTA

19. How many distinct positive integers a exist such that, for some integer $b > 1$, $a^b - 1$ is prime?
A) 0 B) 1 C) 2 D) Infinitely many E) NOTA

20. What (if there is one) is the flaw in the "proof?"
A) No flaw. B) Illegal operations in the modular arithmetic. C) $(a-1) \mid (a^b - 1)$ not always true.
D) The three given factors are not necessarily distinct. E) NOTA

Consider the following proof that 2 = 1:
Given two numbers a and b such that a = b:

Statement	**Justification**
$a^2 = ab$	Multiplying by a.
$-a^2 = -ab$	Multiplying by –1.
$b^2 - a^2 = b^2 - ab$	Adding b^2.
$(b-a)(b+a) = b(b-a)$	Factoring.
$b + a = b$	Dividing by $(b-a)$.
$2b = b$	Substituting b for a, since $a = b$.
$2 = 1$	Dividing by b.

21. Which of the following statements could be shown to be true by applying some combination of the four basic operations to both sides of the "equation" 2 = 1?

I. 1 = 0 **II**. 10 = –10 **III**. 5 = 5 **IV**. $e = \pi$ **V**. Winston Churchill was a carrot.

A) **III** only. B) **I** only C) **I**, **II**, and **III** only D) **I**, **II**, **III**, **IV**, and **V** E) NOTA

22. This truly disastrous result shows the danger of committing what mathematical sin?
A) Factoring improperly B) Plugging in when you're not supposed to.
C) Multiplying by an unknown quantity. D) Dividing by zero. E) NOTA

$$(a+b+c)(d+e+f) = (a+b)(d+e+f) + c(d+e+f) = a(d+e+f) + b(d+e+f) + c(d+e) + cf =$$
$$a(d+e) + af + b(d+e) + bf + cd + ce + cf = ad + ae + af + bd + be + bf + cd + ce + cf$$

23. The justification for every step of the above expansion is the:

A) Binomial Expansion Theorem B) Associative Law of Multiplication
C) Distributive Property D) Associative Law of Addition E) NOTA

24. Auntie Em wants to divide 35 pancakes randomly among 6 Musa's. Consider the following list of statements; assign a value of 2 to each statement that is always true, 1 to each statement that is sometimes (but not always) true, and 0 to statements that are never true.

At least 5 Musa's will each have at least 6 pancakes.
At least 1 Musa will have at least 6 pancakes.
At least 1 Musa will have less than 6 pancakes.
All 6 Musa's will each have at least 5 pancakes.
No one Musa will have more than 30 pancakes.
At least 5 Musa's will each have less than 7 pancakes.

What is the sum of the values of all the statements?
A) 7 B) 8 C) 9 D) 10 E) NOTA

25. All the statements above which are always true are direct results/examples of which of the following?

A) The Four-Color Theorem B) The Pigeonhole Principle
C) Xeno's Paradox D) Archimedes' Distribution Theory E) NOTA