



1. 117649 This is a list of cube numbers where the final digit or 2 digits is their cube root.
2. $7^3 - 6^3 - 3^3 = 100$ and $1870^3 - 1797^3 - 903^3 = 100$
3. It contains all the digits 1-9 in ALPHABETICAL ORDER.
4. 111—These are numbers that read the same right-side up or upside down.
5. 2520 is the lowest number into which all the digits from 1 to 10 will divide evenly.
6. 0, 9, 16; 0, 36, 64; 41, 80, 320
7. a) Between 999 and 1001 with just a 2-year gap. b) The next time will be between 9999 and 10001.
8. 2—The numbers in the 1st 2 horizontal and vertical lines, divided by either 5 or 4, give the figure in the final column or row. For example, $15 \div 5 = 3$ and $8 \div 4 = 2$, so $3 + 2 = 5$ (the last number in that row).
9. Answer: $-27i$
10. Answer: (0, 0, 0) (1, 1, 1), (1, -1, -1), (-1, 1, -1), (-1, -1, 1)
11. CXLIV
12. thrown out
13. The numbers are 8, 13, 24, 25, 34, 34, 81, 101.
 - Clue 7. The largest number is the smallest 3 digit palindrome: 101
 - Clue 1. The range, maximum - minimum, is $101 - 8 = 93$, so the smallest number must be 8.
 - Clue 3. A cube less than 10: that would have to be $8=2^3$ or $1=1^3$. This clue is not really needed because of Clue 1.
 - Clue 4. The arithmetic mean of 8 numbers is 40, so the sum of the numbers is $8*40=320$.
 - Clue 2. The first few abundant numbers (numbers such that the sum of the divisors less than the number is greater than the number) are 12, 18, 20, 24, 30, ... hence the abundant number between 21 and 25 would have to be 24.
 - Clue 5. The mode of the list is the sum of the first two perfect numbers, numbers such that the sum of the proper divisors equal the number. The first two are 6 and 28, hence the mode is 34. Since a mode would not be defined unless at least two numbers in the list are the same, we assume two number 34s.
 - Clue 6. The median is $29.5 = \text{mean of two successive entries, one of which is the maximum of the lower four numbers and one of which is the minimum of the upper four numbers, e.g., 29 and 30, or 28 and 31, or 27 and 32, or 26 and 33, or 25 and 34. Aha! We've got it: 25 and 34.}$
 - Clue 8. Looking at a table of squares, the only one under 121 such that the sum of its digits is the root is 81. . . "Summing up"; from least to greatest we have: 8, x, 24, 25, 34, 34, 81, 101. x is a slack variable, chosen so that the sum of the numbers is 320, that is $x=13$.



14. Answer 1,000

If you computer the first four sums 1, 8, 27, 64, you can perhaps guess that the sum of the i^{th} group is i^3 . One way to prove it is to note that the i^{th} group has i numbers and that their average is i^2 . To see this when i is odd, note that there will

be $i(i-1)/2$ odd numbers preceding the group, and the middle entry will be the $\left(\frac{i+1}{2}\right)^{\text{th}}$ in the group. It will thus equal

$$-1 + \frac{2i(i-1)}{2} + \frac{i+1}{2} = i^2. \text{ A similar argument will work if } i \text{ is even.}$$

15. The shortest route is 40 feet. The spider crawls along 5 of the 6 sides of the room.

16. 42 minutes

17. Sunday

18. Tuesday

19. $10!$ days or 3628800 days.

20. The banana is 5.75 inches long.

Let A = age of the monkey now and let M = age of his Mother. Then $A + M = 30$. From the long sentence, $M = .5(3(.5(4(2((1/3)(3(.25A))))))$ Solving simultaneously, $2.5 A = 30$, so $A = 12$ (the age of the monkey) and $M = 18$ (the age of the monkey's mother). So the weight of the monkey is 18 oz. The length of the rope is 12 feet. The rope weighs 2 pounds. The weight weighs 18 ounces. The banana weighs 11.5 ounces, so the banana is 5.75 inches long.

21. ONE OCTILLION

22. Jack is 26 & Diane is 20.

Break the problem into 2 parts: Part 1 (before the comma)--Let $D1$ = Diane's age when Jack was $\frac{1}{2}$ as old as he is now $= J/2 - (J - D) = D - J/2$. Let $D2$ = Diane's age when Jack was twice as old as $D1 = 2D1 - (J - D) = 2D - J - (J - D) = 3D - 2J$.

Part 2 (after the comma)—Let $J1$ = Jack's age when Diane was a year older than $\frac{1}{2}$ as old as he is now $= (J/2 + 1) + (J - D) = 3J/2 - D + 1$. So, Diane's age for part 2 is $\frac{1}{2}(J1)$, which is $\frac{1}{4}(3J - 2D + 2)$. Now we know that when jack was $6D - 4J$ years old, Diane was $\frac{1}{4}(3J - 2D + 2)$ years old, which creates the equation: $(6D - 4J) - (J - D) = \frac{1}{4}(3J - 2D + 2)$, which simplifies to: $30D = 23J + 2$. From this, we deduce that Jack's age must end in a 6, and the only number that ends in 6 in the twenties is

26. Either Bono or Edge returns

23. $7^2 = 49$ (the 6 is turned over to make it a 9)

24. $\frac{1}{3^{32}}$ or $\frac{1}{1,853,020,188,851,841}$. There are 32 warts on the frog and each wart can be one of 3 possible colors.

This gives us 3^{32} combinations and only one of those is all purple.

25. 2:56 PM--Because Cutty starts with engines, the 108th engine will occur during his fifth batch of items, for a total of 208. At 2 minutes per item, he will complete his 108th engine 416 minutes after he starts building.

26. Send Bono over with Edge—2 min; then Bono returns—1 min; then Adam and Larry go over—10 min; then Edge returns—2 min; and last Bono and Edge go over—2 minutes. This gives a total of 17 minutes.

27. $m\angle 1 = 110^\circ$; $m\angle 2 = 70^\circ$; $m\angle 3 = 55^\circ$; $m\angle 4 = 55^\circ$; $m\angle 5 = 70^\circ$; $m\angle 6 = 20^\circ$; $m\angle 7 = 160^\circ$; $m\angle 8 = 20^\circ$; $m\angle 9 = 70^\circ$; $m\angle 10 = 90^\circ$; $m\angle 11 = 20^\circ$; $m\angle 12 = 70^\circ$; $m\angle 13 = 90^\circ$; $m\angle 14 = 90^\circ$; $m\angle 15 = 90^\circ$; $m\angle 16 = 90^\circ$; $m\angle 17 = 90^\circ$; $m\angle 18 = 55^\circ$; $m\angle 19 = 70^\circ$; $m\angle 20 = 55^\circ$; $m\angle 21 = 55^\circ$; $m\angle 22 = 70^\circ$; $m\angle 23 = 55^\circ$; $m\angle 24 = 35^\circ$; $m\angle 25 = 145^\circ$; $m\angle 26 = 35^\circ$



28. 7 ounces

Let x = the peeled banana
Let y = the banana peel
So, $x + y$ = the unpeeled banana

From the first panel of the comic strip, $y = \frac{1}{8}(x+y)$

From panel 2: $x+y = x + \frac{7}{8}$

So, $y = \frac{7}{8}$ oz

From step 1: $y = \frac{1}{8}x + \frac{1}{8}y$

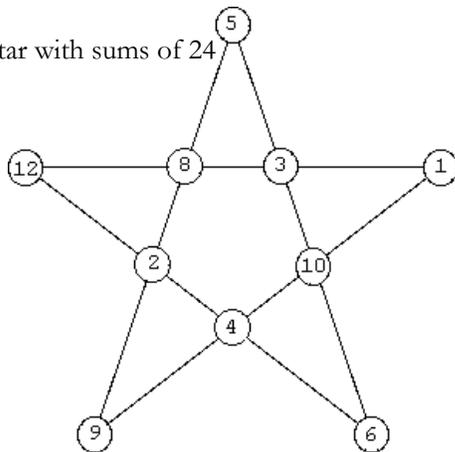
Multiplying by 8: $8y = x + y$

Therefore, $7y = x$

Substituting for y : $x = 7\left(\frac{7}{8}\right) = \frac{49}{8}$

Finally, the unpeeled banana equals 7 oz $x+y = 7$ oz

29. Any star with sums of 24



30. Answer: If I have seen further it is by standing on the shoulders of giants. Sir Isaac Newton