



1. **D.**
2. **D.**
3. **B.**
4. **B.**
5. **C.**
6. **A.**
7. **E.**
8. **C.**
9. **A.**
10. **D.**
11. **D.**
12. **C.**
13. **THROWN OUT**
14. **B.**
15. **D.**
16. **B.**
17. **C.**
18. **C.**
19. **A.**
20. **D.**
21. **E.**
22. **C.**
23. **A.**
24. **B.**
25. **C.**
26. **D.**
27. **A.**
28. **B.**
29. **B.**
30. **C.**



1. **D.** 13
2. **D.**  $(1000+1)^2 - (1000-1)^2 = [(1000+1)+(1000-1)][(1000+1)-(1000-1)] = [2000][2] = 4000$
3. **B.** for example-  $\{2, \pi\}$  2: complex, rational, integer, real, natural  $\pi$ : irrational, transcendental
4. **B.** (from 10.5) No, all perfect squares have an odd number of positive integer divisors because all factors come in pairs, except when the factor multiplies by itself.
5. **C.** The answer is taken by using appendix A's clue. Problem numbers that are bolded (2, 5, 13, 14, 18, 19, 21) correspond to the letters that unscramble to the word NUMBERS.
6. **A.** The first answer choice is correct. Test-takers should take note of choice D for future reference.
7. **E.**  $2+5+3-2-5-3+2+ \dots 2+5+3-2$ . Terms 1 through 96 cancel out, so summing last 4 terms = 8.
8. **C.** Exterior angles (1 at each vertex) must add up to  $360^\circ$ . With 360 sides (and 360 vertices), each exterior angle must be  $1^\circ$ . Minimal integer value for exterior angles maximizes integer value for interior angles.
9. **A.** Columns are Pythagorean triples.  $28^2 + 45^2 = 53^2$
10. **D.**  $5^2 - 1 = 24$ ;  $7^2 - 1 = 48$ . Simple testing of primes yields 24.  
Alternatively: let  $p$  be a prime number  $>3$ .  $p^2 - 1 = (p-1)(p+1)$ .  $p+1$  and  $p-1$  must both be even, and since alternating evens are multiples of 4, one must also be a multiple of 4. Since  $p-1, p, p+1$  are 3 consecutive numbers, one of them must be a multiple of 3. Since  $p$  is prime, either  $p-1$  or  $p+1$  must be a multiple of 3. Thus,  $(p-1)(p+1)$  must have factors  $2*4*3=24$ .
11. **D.** Directions at beginning of test state what to do for a question that does not exist.
12. **C.** 1331 is a palindrome, 6889 is an ambigram, 2187 is a vampire (its fangs are 27 and 81)
13. **A.** The sum of the first ten primes is 129.  $129-8=121$ . Both 11 and -11 are answers. **The answer key will be changed to accept both A and B.**
14. **B.** More than 3 acute angles and the polygon must be concave.
15. **D.**  $1111+8888+9999=19998$
16. **B.** Aaron (or Baron) can build 2 houses in 8 days. To get to 14 houses, it takes  $8*7=56$  days.
17. **C.** The problem boils down to finding a 4 digit number whose digits are reversed when multiplied by 4. Through logical processes, the number 21978 can be found.
18. **C.**  $1+2+\dots+n = (n+1)(n)/2$ . For  $\sqrt{n(n+1)/2}$  to be an integer, one of the two numbers  $n$  and  $(n+1)$  must be a perfect square, and the other must be twice a perfect square. The only consecutive numbers that fit that description are 8 & 9 and 49 & 50. So  $n=8$  or  $n=49$ .
19. **A.** Zero chords divide it into 1 part. One chord gives 2 parts. Two chords give 4. Three chords give 7. Four chords give 11.  $n$  chords divide it into  $n(n+1)/2 + 1$  parts.
20. **D.** The least powers of two that are integers and fit this restriction are  $a=2^1=2, b=2^0=1, c=2^1=2. 2+1+2=5$ .
21. **E.**  $10 \bullet 63 = 6 \bullet 57 + 4 \bullet a \Rightarrow 630 - 342 = 4a \Rightarrow 288 = 4a \Rightarrow a = 72$
22. **C.** Note that  $\frac{6}{11} = \frac{36}{66}, \frac{5}{22} = \frac{15}{66},$  and  $\frac{4}{33} = \frac{8}{66}$ . Finding the LCM of 36, 15, and 8 and then dividing by 66 gives the answer. LCM of 36, 15, and 8 is 360.  $360/66 = 60/11$ .
23. **A.** Prime factorization of  $120 = 2^3*3*5$ . The expansion of  $(2^3 + 2^2 + 2^1 + 2^0)(3^1 + 3^0)(5^1 + 5^0)$  gives the sum of all the factors of 120. Instead of expanding the product, simplification gives  $(15)(4)(6)=360$ .
24. **B.** Simply looking at the first two-digit prime shows that 11 is a palindrome. All other palindromes with an even number of digits have a factor of 11.
25. **C.** A cute number has only two factors (other than one and itself). Those among the first 25 positive integers are: 6, 8, 10, 14, 15, 21, and 22.
26. **D.** Note that the most important aspect of the numbers is their exponent. The number with the largest exponent is clearly  $2^{2^{22}}$ .
27. **A.** Let the prime factorization of  $n$  be  $2^P 3^Q 5^R$ . Since  $2n$  is a perfect square,  $P$  must be odd and  $Q$  and  $R$  must be even. Since  $3n$  is a perfect cube,  $Q$  must be one less than a multiple of 3 and  $P$  and  $R$  must be multiples of 3. Since  $5n$  is a perfect fifth power,  $R$  must be one less than a multiple of 5 and  $P$  and  $Q$  must be multiples of 5. So:  $P$  is a multiple of 5 and 3 (thus a multiple of 15) and one less than a multiple of 2; this gives  $P=15$  as the least value.  $Q$  is a multiple of 2 and 5 (and thus 10) and is one less than a multiple of 3; this gives  $Q=20$  as the least value.  $R$  is a multiple of 2 and 3 (and thus 6) and one less than a multiple of 5; this gives  $R=24$ . So  $n = 2^{15}3^{20}5^{24}$
28. **B.** For  $16!/n$  to be a perfect square, its prime factorization must have only even powers. Looking at the prime factorization of  $16!$  shows that 2, 5, 11, and 13 all have odd exponents in the prime factorization. Dividing  $16!$  by  $(2*5*11*13)$  gives a perfect square, thus  $n=2*5*11*13=1430$



29. **B.** Each decimal place from the units digit to the hundred-millions digit can be taken up by any digit 0 through 9 (one-digit numbers can be represented as 000,000,001, 000,000,002, etc because we are summing up digits, so 0 does not affect the total). With that in mind, we know that since there are 10 digits and 9 different decimal places, each digit gets used  $10^8=100,000,000$  times in each decimal place and thus is used 900,000,000 times total.  $900,000,000(0+1+2+3+4+5+6+7+8+9)=40,500,000,000$ . Since 1 billion is included, we must add 1 to the total, giving 40,500,000,001.
30. **C.** since  $2*5=10$ ,  $2^{1990}*5^{1991}=10^{1990}*5$ , which is 5 followed by 1990 zeroes. Thus, the sum of its digits is 5.