1. $A = x^3 - x^2 + 2x\Big|_3^1 = 2 - 24 = -22$; $B = 3x^2 - 2x + 1\Big|_{x=2} = 9$; $C = \dfrac{-66}{6} = -11$;

$f'(x) = -2x + 2 = 0 \Rightarrow x_{\max} = 1 \Rightarrow D = f(1) = 5$

$$\begin{vmatrix} -22 & 9 \\ -11 & 5 \end{vmatrix} = (-22)(5) - (-11)(9) = -11$$

2. (A) is true because $f''(x) = 6ax + 2b = 0$ gives $x = -\dfrac{b}{3a}$. (B) is false because

$f'(x) = 3ax^2 + 2bx + c = 0$ gives $x = \dfrac{-2b \pm \sqrt{4b^2 - 2(3a)(c)}}{2(6a)} = -\dfrac{b}{3a} \pm \dfrac{\sqrt{b^2 - 3ac}}{3a}$, so the

horizontal distance between the two solutions for x is $2 \cdot \dfrac{\sqrt{b^2 - 3ac}}{3a}$. (C) is true because

$f(-1) = f'(-1) = 0 \Leftrightarrow -a + b - c + d = 3a - 2b + c \Leftrightarrow 4a - 3b + 2c - d = 0$. (D) is false because for the first of the two extrema to be a local minimum, the curve must first be concave up then concave down, so $f''(x) = 6ax + 2b$ must change from positive to negative and hence have a negative slope $6a$.

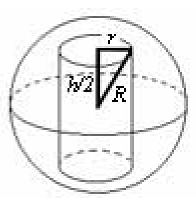
3. $A = \pi\displaystyle\int_0^1 \left[(y+2)^2 - (2+2)^2\right]dx = \pi\int_0^1 \left[(2x+5)^2 - 4^2\right]dx = \pi\int_0^1 \left(4x^2 + 20x + 9\right)dx$

$= \pi\left(\dfrac{4}{3}x^3 + 10x^2 + 9x\Big|_0^1\right) = \dfrac{61\pi}{3}$

$B = 2\pi\displaystyle\int_0^1 (y-2)\cdot(x+2)dx = 2\pi\int_0^1 (2x+1)(x+2)dx = 2\pi\int_0^1 (2x^2 + 5x + 2)dx$

$= 2\pi\left(\dfrac{2}{3}x^3 + \dfrac{5}{2}x^2 + 2x\Big|_0^1\right) = \dfrac{31\pi}{3} \Rightarrow A - B = 10\pi$

4.



$r^2 + \dfrac{h^2}{4} = R^2 \Rightarrow r^2 = R^2 - \dfrac{h^2}{4} \Rightarrow V = \pi r^2 h = \pi\left(R^2 h - \dfrac{h^3}{4}\right) \Rightarrow \dfrac{dV}{dh} =$

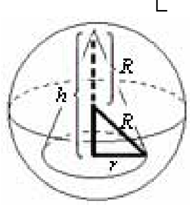
$\pi\left(R^2 - \dfrac{3}{4}h^2\right) = 0 \Rightarrow h = \dfrac{2R}{\sqrt{3}} \Rightarrow V_{\max} = \pi\left[R^2\left(\dfrac{2R}{\sqrt{3}}\right) - \dfrac{8R^3}{12\sqrt{3}}\right] = \dfrac{4\pi}{3\sqrt{3}}R^3 \Rightarrow$

$\alpha = \dfrac{4\pi}{3\sqrt{3}} = \dfrac{4\pi\sqrt{3}}{9}$

5. $\ln f(x) = 3\ln\left(3x^4 + 7\right) + 6\ln\left(2 - 5x\right) + 5\ln\left(\cos\left(x - \dfrac{\pi}{6}\right)\right) \Rightarrow \dfrac{f'(x)}{f(x)} = \dfrac{36x^3}{3x^4 + 7} - \dfrac{30}{2 - 5x} - 5\tan\left(x - \dfrac{\pi}{6}\right)$

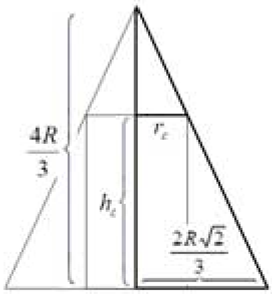
$f'(0) = f(0)\left[-\dfrac{30}{2 - 5(0)} - 5\tan\left(-\dfrac{\pi}{6}\right)\right] = (7)^3(2)^6\cos^5\left(-\dfrac{\pi}{6}\right)\left(-15 + \dfrac{5}{\sqrt{3}}\right) = 30870 - 92610\sqrt{3}$

6.



$r^2 + (R - h)^2 = R^2 \Rightarrow r^2 = 2Rh - h^2 \Rightarrow V = \dfrac{\pi}{3}r^2 h = \dfrac{\pi}{3}\left(2Rh^2 - h^3\right)$

$\dfrac{dV}{dh} = \dfrac{\pi}{3}\left(4Rh - 3h^2\right) = 0 \Rightarrow h = \dfrac{4R}{3} \Rightarrow r = \dfrac{2R\sqrt{2}}{3}$



$$\frac{4R/3-h_c}{r_c}=\frac{4R/3}{2R\sqrt{2}/3}=\sqrt{2}\Rightarrow h_c=\frac{4R}{3}-r_c\sqrt{2}$$

$$V_c=\pi r_c^{\,2}h_c=\pi\left(\frac{4R}{3}r_c^{\,2}-r_c^{\,3}\sqrt{2}\right)\Rightarrow\frac{dV_c}{dr_c}=\pi\left(\frac{8R}{3}r_c-3r_c^{\,2}\sqrt{2}\right)=0$$

$$\Rightarrow r_c=\frac{4R\sqrt{2}}{9}\Rightarrow V_{c\max}=\pi\left[\left(\frac{4R\sqrt{2}}{9}\right)^2-\left(\frac{4R\sqrt{2}}{9}\right)^3\sqrt{2}\right]=\frac{128\pi}{729}R^3$$

$$\beta=\frac{128\pi}{729}$$

7. $x^3-2x^2-9x+18=(x-2)(x-3)(x+3)\Rightarrow\int_{-5}^{5}\left|x^3-2x^2-9x+18\right|dx=\int_{-5}^{-3}-(x^3-2x^2-9x+18)dx$

$$\int_{-5}^{5}\left|x^3-2x^2-9x+18\right|dx=\int_{-5}^{-3}-(x^3-2x^2-9x+18)dx+\int_{-3}^{2}(x^3-2x^2-9x+18)dx$$

$$=\left(-\frac{1}{4}x^4+\frac{2}{3}x^3+\frac{9}{2}x^2-18x\right|_{-5}^{-3}+\left(\frac{1}{4}x^4-\frac{2}{3}x^3-\frac{9}{2}x^2+18x\right|_{-3}^{2}=\frac{280}{3}+\frac{875}{12}=\frac{665}{4}$$

8. The graph is of a semicircle of radius 6 lying on the *x*-axis. $M=$ area of semicircle $=\frac{1}{2}\pi(6)^2=18\pi$;

$A=$ area of sector of angle $\frac{\pi}{2}-\theta=\frac{\pi/2-\theta}{\pi}\cdot 18\pi=9\pi-18\theta$; $R=$ length of sector of angle

$\left(\frac{5\pi}{6}-\frac{\pi}{4}\right)=6\left(\frac{5\pi}{6}-\frac{\pi}{4}\right)=\frac{7\pi}{2}$; $K=$ slope of tangent to circle at a radius of slope $b=-\frac{1}{b}$

$$MARK=(18\pi)(9\pi-18\theta)\left(\frac{7\pi}{2}\right)\left(-\frac{1}{b}\right)=\frac{1134\pi^2\theta-567\pi^3}{b}$$

9. Let $f(x)=Ax^3+Bx^2+Cx+D$. Substituting the information from the problem into $f(x)$, $f'(x)$, and $f''(x)$ gives $f''(2)=12A+2B=0\Rightarrow B=-6A$; $f'(-1)=3A-2B+C=0\Rightarrow C=-15A$; $f(2)=8A+4B+2C+D=-46A+D=-6$ and $f(-1)=-A+B-C+D=8A+D=48$.
This system in A and D has solution $(A,D)=(1,40)\Rightarrow B=-6\Rightarrow C=-15$.
$A+B+C+D=1-6-15+40=20$

10. $\int_0^A we^{w^2}dw=\frac{1}{2}e^{w^2}\Big|_0^A=\frac{1}{2}\left(e^{A^2}-1\right)=9\Rightarrow A=\sqrt{\ln 19}$

$$\int_1^B y\ln y\,dy=\left(\frac{1}{2}y^2\ln y-\frac{1}{4}y^2\right|_1^B=\frac{1}{2}B^2\ln B-\frac{1}{4}B^2+\frac{1}{4}=\frac{1}{4}+\frac{1}{4}e^2\Rightarrow B^2(2\ln B-1)=e^2\Rightarrow B=e$$

$A+B=\sqrt{\ln 19}+e$

11. $f'(x)=2x-2=0\Rightarrow x=1=A$; $f'(x)=2x-2=m_{[6,12]}=\frac{112-16}{6}=16\Rightarrow x=9=B$;

$$C=\frac{1}{15}\int_{-6}^{9}(x^2-2x-8)dx=\frac{1}{15}\left(\frac{1}{3}x^3-x^2-8x\right|_{-6}^{9}=\frac{1}{15}\left[90-(-60)\right]=10;$$

$f'(x)=2x-2=0\Rightarrow x=1\Rightarrow$ min $=f(1)=-9=D$. $A+B+C+D=1+9+10-9=11$

12. The first and fourth statements are true and the fifth statement is false from direct substitution. The third is true because $f'(x)=6(2x^3-5x^2-9x+18)\Rightarrow f'(-2)=6(-16-20+18+18)=0$, $f(-2)=48+80-324-74=-270$, and $f''(x)=6(6x^2-10x-9)\Rightarrow f''(-2)=6(24+20-9)>0$. The second is false because $f''(2)=6(24-20-9)\neq 0$. This gives $\{-4,0,5\}$ for the values of the true statements, so their sum is $-4+0+5=1$.

13. $$\int_{\frac{3}{2}}^{\frac{9}{4}}\frac{dx}{\sqrt{3x-x^2}}=2\int_{\frac{3}{2}}^{\frac{9}{4}}\frac{1/2\sqrt{x}}{\sqrt{3-x}}dx=2\arcsin\sqrt{\frac{x}{3}}\Bigg|_{\frac{3}{2}}^{\frac{9}{4}}=2\left(\arcsin\frac{\sqrt{3}}{2}-\arcsin\frac{\sqrt{2}}{2}\right)=2\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\frac{\pi}{6}$$

14. Each of the limits is simply the definition of derivative.

$$\frac{d}{dx}(\ln x)=\frac{1}{x}\Rightarrow A=f'(1)=\frac{1}{1}=1$$

$$\frac{d}{dx}(\cot x)=-\csc^2 x\Rightarrow B=g'\left(\frac{\pi}{6}\right)=-\csc^2\left(\frac{\pi}{6}\right)=-4$$

$$\frac{d}{dx}\left(x^3+2x^2+3x+4\right)=3x^2+4x+3\Rightarrow C=h'(3)=3(3)^2+4(3)+3=42$$

$$A+B+C=1-4+42=39$$

15. Use integration by parts twice, first with $u=e^{4x}$ and $dv=\cos 5x\,dx$, then with $u=e^{4x}$ and $dv=\sin 5x\,dx$. This gives $\int e^{4x}\cos 5x\,dx=\frac{1}{5}e^{4x}\sin 5x-\frac{4}{5}\left(-\frac{1}{5}e^{4x}\cos 5x+\frac{4}{5}\int e^{4x}\cos 5x\,dx\right)$.

Rearranging gives $\int e^{4x}\cos 5x\,dx=\frac{e^{4x}}{41}(5\sin 5x+4\cos 5x)$. Evaluating the definite integral gives

$$\frac{e^{5\pi}}{41}\left(5\sin\frac{25\pi}{4}+4\cos\frac{25\pi}{4}\right)-\frac{e^{2\pi/3}}{41}\left(5\sin\frac{5\pi}{6}+4\cos\frac{5\pi}{6}\right)=\frac{9e^{5\pi}\sqrt{2}-5e^{2\pi/3}+4e^{2\pi/3}\sqrt{3}}{82}.$$