

Alpha Logs and Exponents
2008 National Convention

1. C

2. A

3. B

4. B

5. C

6. A

7. C

8. D

9. B

10. A

11. A

12. C

13. B

14. E (1)

15. B

16. D

17. A

18. C

19. A

20. C

21. B

22. E (-448)

23. E (undefined)

24. C

25. A

26. B

27. B

28. D

29. B

30. C

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1. $\log \frac{4x^{\frac{1}{2}}}{y^2} = \log \frac{4\sqrt{x}}{y^2}$ **C**

$$e^x = 5e^x - 5$$

2. $-4e^x = -5$ **A**

$$e^x = \frac{5}{4}$$

$$x = \ln \frac{5}{4}$$

3. $90 = 75 + 5e^{\frac{\ln^3 t}{8}}$ **B**

$$3 = e^{\frac{\ln^3 t}{8}}$$

$$t = 8$$

4. $1.5P = Pe^{-1t}$ **B**

$$1.5 = e^{-1t}$$

$$\frac{\ln 1.5}{-0.1} = t$$

5. $9^3 = 3^2 9^2$ **C**

$$2 + 3 = 5$$

6. $\frac{1}{3}[\log_b 4 + 5 \log_b x + 7 \log_b y]$ **A**

7. *characteristic* = 2 **C**

$$\textit{mantissa} = 0.6385$$

$$20 = 5e^{4r}$$

$$4 = e^{4r}$$

8. $r = \frac{\ln 4}{4}$ **B**

$$P = 5e^{24\left(\frac{\ln 4}{4}\right)} = 5(4096) = 20,480$$

9. $y = 2e^{-0.21t}$ **B**

$$f(x) = -\log_3(x+2) - 4$$

10. $x + 2 = 0$ **A**

$$x = -2$$

$$a. 2000e^5$$

11. $b. 2000\left(1 + \frac{.05}{4}\right)^{40} = 2000(1.0125^{40})$ **A**

$$c. 2000\left(1 + \frac{1}{2}\right)^{20} = 2000(1.05^{20})$$

$$d. 2000\left(1 + \frac{1}{12}\right)^{120} = 2000(1.008)^{120}$$

12. Degree of Denominator = Degree of Numerator, use the ratio of the coefficients: **C**

$$\frac{300,000}{1,000} = \$300$$

$$13. \left(\frac{-4!uv^{-20}v^u}{8! \frac{u^{-8}v^{-8}u^{-80}}{v^{2008}}} \right)^2 = \left(\frac{-uv^{-20+u}}{1680u^{-88}v^{-2016}} \right)^2 = \left(\frac{-u^{89}v^{1996+u}}{1680} \right)^2 = \frac{u^{178}v^{3992+2u}}{1680^2} \quad \mathbf{B}$$

$$14. \text{Arc cos}(1) = 0 \quad \mathbf{E (NOTA)}$$

$$\left(\frac{(n-1)^{-2}}{(n^2-1)^{-3}} \right)^0 = 1$$

$$15. \frac{(\log 3^5)(\log 5^4)(\log 6^3)}{(\log 6^2)(\log 3^6)(\log 5^2)} = \frac{(5 \log 3)(4 \log 5)(3 \log 6)}{(2 \log 6)(6 \log 3)(2 \log 5)} = \frac{60}{24} = \frac{5}{2} \quad \mathbf{B}$$

$$16. \begin{aligned} & 3^{-x}(9^{2x^2}(27^{-7x}(243^{-\frac{2}{5}}))) \\ & 3^{-x}(9^{2x^2}((3^3)^{-7x}(3^{-2}))) \\ & 3^{-x}(9^{2x^2}((3^{-21x})(3^{-2}))) \quad \mathbf{D} \end{aligned}$$

$$\begin{aligned} & 3^{-x}((3^2)^{2x^2}((3^{-21x-2}))) \\ & 3^{-x}((3^{4x^2})((3^{-21x-2}))) = 3^{4x^2-22x-2} \\ & 2^8 \approx 250, 3^5 \approx 250 \\ & 2^{2008} \geq 250^{251}, 3^{1004} \approx 250^{201} \end{aligned}$$

$$17. \frac{1}{8}^{-502} = 2^{-3(-502)} = 2^{1506} \quad \mathbf{A}$$

$$\begin{aligned} & \frac{1}{9}^{-251} = 3^{-2(-251)} = 3^{502} \\ & 2 \cos^2 x - \cos x - 1 = 0 \\ & (2 \cos x + 1)(\cos x - 1) = 0 \end{aligned}$$

$$18. \begin{aligned} & \cos x = \frac{-1}{2}, 1 \quad \mathbf{C} \\ & 0 < x \leq \pi \\ & \cos x \neq 1 \end{aligned}$$

$$\begin{aligned} & \left(\frac{5\pi}{6}, \frac{-1}{2} \right) \\ & (2^6)^{-x} = 2^{10} \end{aligned}$$

$$19. -6x = 10 \quad \mathbf{A}$$

$$\begin{aligned} & x = \frac{-5}{3} \\ & 4x^2 + 43x - 62 = 2 \end{aligned}$$

$$20. \begin{aligned} & 4x^2 + 43x - 64 = 0 \quad \mathbf{C} \\ & \frac{c}{a} = \frac{-64}{4} = -16 \end{aligned}$$

$$(1-i)$$

$$\text{Radius} = \sqrt{2}, \text{Angle} = \frac{-\pi}{4}$$

21. $(\sqrt{2})^{24} \text{cis} 24\left(\frac{-\pi}{4}\right) = 2^{12} \text{cis}(-6\pi)$ **B**

$$2^{12}(\cos(-6\pi) + i \sin(-6\pi)) = 2^{12}(1+0i) = 2^{12}$$

22. $C(8,3)(2x)^3(-y)^5$ **E = -448**
 $-448x^3y^5$

$$\frac{\sin^2 x \sec^2 x}{1 + \tan^2 x} = \frac{\sin^2 x \sec^2 x}{\sec^2 x}, \sin^2 x = 1$$

23. $\sin x = \pm 1$ **Both values make equation undefined. E**

$$x = \pm \frac{\pi}{2}$$

undefined

$$e^{i\theta} = \text{cis}\theta$$

24. $\text{cis} \frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}i}{2}$ **C**

25. $\left(-\frac{i}{2}\right)^{2008} = \left(\frac{-1}{2}\right)^{2008} i^{2008}$ **A**

$$\frac{1}{2^{2008}}(1) = \frac{1}{2^{2008}}$$

$$6^x(6^{2x}) = 54$$

26. $6^{3x} = 54$ **B**
 $3x \log 6 = \log 54$

$$x = \frac{\log 54}{3 \log 6}$$

27. $\left(x^{\frac{2}{3}} - 27\right)\left(x^{\frac{2}{3}} - 64\right) = 0$ **B**

$$x^{\frac{2}{3}} = 27, 64$$

$$x = 27^{\frac{3}{2}}, 64^{\frac{3}{2}}$$

$$\sum x = 27^{\frac{3}{2}} + 64^{\frac{3}{2}}$$

$$8x^2(2x-1) + 15(2x-1) \text{grouping}$$

28. $(8x^2 + 15)(2x - 1) > 0$ **D**

$$8x^2 + 15 \neq 0, 2x - 1 = 0, x = \frac{1}{2}$$

$$x > \frac{1}{2}$$

$$x \begin{pmatrix} 0 & \frac{1}{5} \\ 25 & -1 \end{pmatrix} - 5 \begin{pmatrix} 81 & \frac{1}{5} \\ e^5 & -1 \end{pmatrix} + \begin{pmatrix} 81 & 0 \\ e^5 & 25 \end{pmatrix} = 0$$

$$29. \quad -5x - 5[-81 - e^5] + 2025 = 0$$

B

$$-5x + 405 + 5e^5 + 2025 = 0$$

$$-5x + 5e^5 = -2430$$

$$x = 486 + \frac{e^5}{5}$$

$$30. \quad \left[\frac{(x^2 - y^2)^{-3}}{-(x+y)^{-4}} \right]^{\frac{-2}{3}} = \left[\frac{-(x+y)^4}{(x^2 - y^2)^3} \right]^{\frac{-2}{3}} = \left[\frac{-(x+y)(x+y)(x+y)(x+y)}{(x-y)(x+y)(x-y)(x+y)(x-y)(x+y)} \right]^{\frac{-2}{3}} \quad \mathbf{C}$$

$$\left[\frac{-(x+y)}{(x-y)(x-y)(x-y)} \right]^{\frac{-2}{3}} = \left[\frac{(x-y)^3}{-(x+y)} \right]^{\frac{2}{3}} = \frac{(x-y)^2}{(x+y)^{\frac{2}{3}}}$$