1. **A** The product of the GCD and LCM of any two numbers is just the product of the numbers, so 56 * 72 = 4032.
2. **D.** There are infinite pairs, such as $(\sqrt[4]{2},\sqrt[4]{3},\sqrt[4]{5})$, $(\sqrt[4]{2},\sqrt[4]{4},\sqrt[4]{6})$, $(\sqrt[4]{2},\sqrt[4]{5},\sqrt[4]{7})$…
3. **C.** 49,504 = $2^5*7*13*17$, so there are 6*2*2*2 factors since in any factor there must either be one or no 7's, one or no 13's, one or no 17's, five, four, three, two, one or no 2's. But for the factor to be even, it can't have no 2's, so there are only 5*2*2*2 even factors. This means that the probability of a randomly chosen factor being even is $5/6$.
4. **B.** Solutions are $4+7(0)$, $4+7(1)$ … $4+7(107)=753$, so there are 108 in total.
5. **D** To convert a binary number to hex, you just have to convert it in groups of four bits since every four bits expresses a number between 0 and 15, just like one digit in hex. $1101_2 \to D_{16}$, $0100_2 \to 4_{16}$, $1000_2 \to 8_{16}$, $1001_2 \to 9_{16}$ and $D_{16}+4_{16}+8_{16}+9_{16}=22_{16}$
6. **C** Proving this remains one of the most elusive open problems in Number Theory today.
7. **B.** Since y has 25 factors, either $y=a^4b^4$ for primes a,b or $y=c^{24}$ for some prime c. This means $y^2=a^8b^8$ or $y^2=c^{48}$ so y^2 has 81 or 49 factors, so 130.
8. **A.** Since each player must lose exactly once, other than the sole winner, there must be 96 losses, which means 96 games played, so 15.
9. **A.** $94_b > 124_b \to 9b+4 > b^2+2b+4 \to 7b > b^2 \to 7 > b$, but if $7 > b$, there can't be any 9 digit, so there are no valid values of b.
10. **B.** 1584's prime factors are 2, 3 and 11. The probability that a number less than 1584 isn't divisible by 2 and isn't divisible by 3 and isn't divisible by 11 is $1/2 * 2/3 * 10/11 = 10/33 > 3/10$, so it is closest to 0.4.
11. **E (27+64*k).** Since $\Phi(64)=32$, $3^{32}\equiv 1 \bmod 64 \to 3^{96}\equiv 1 \bmod 64 \to 3^{99}\equiv 27 \bmod 64$.
12. **B.** Since $1920=2^7*3*5$ must have a power of 2 up to 7, a factor of 3 up to 1 and a factor of 5 up to 1, so $(2^0+2^1+2^2+2^3+2^4+2^5+2^6+2^7)(3^0+3^1)(5^0+5^1)$ will generate a sum containing all of 1920's factors. This expression is equivalent to
$$\frac{2^8-1}{2-1}*\frac{3^2-1}{3-1}*\frac{5^2-1}{5-1}=\frac{2^8-1}{1}*\frac{8}{8}*24=2^8*2^2*6-24=1024*6-24=6120$$
13. **D.** We have to find out how many 5's are in 123! First there are $\lfloor 123/5 \rfloor = 24$ numbers multiplied in 123! that have a power of 5. $\lfloor 123/25 \rfloor = 4$ of the numbers have two 5's. The total number of 5's in 123! is 24+4 = 28.
14. **B (2010).** $x=16072-\frac{8y}{5}$. The smallest value of y that gives an integer x is $y=0, x=16072$. In order for x to remain an integer, y must increase by 5's, so that decreases x by 8. This means that 16072, 16064, 16056… 0 are all acceptable values of x, so in other words, 2009(8), 2008(8)… 0(8), so there are 2010 solutions.
15. **B.** First note that these three statements are equivalent to saying: $x\equiv -2 \bmod 2$, $x\equiv -2 \bmod 7$, $x\equiv -2 \bmod 11$, so $x\equiv -2 \bmod 154$ since 2, 7 and 11 are relatively prime. The 5 smallest positive solutions are: $154(1)-2$, $154(2)-2$, $154(3)-2$, $154(4)-2$, and $154(5)-2$. So the sum is $154(15)-2(5)=2200$.

16. **A. Euclid**
17. **C.** This question is asking for the largest n that cannot be written as $n = 9x + 9y + 16z$ for integer x, y, z, but that's equivalent to $n = 9(x + y) + 16z$ or $n = 9w + 16z$, since $(x + y)$ will always be an integer and can be any integer. From here, it's just a coin problem of order two, so the largest $n = 9*16 - 16 - 9 = 119$.
18. **C.** Since 50! = 50 * 49 * 48 * … , all the primes less than 50 will be in its factorization. Furthermore, assume there is some prime p|50!, p > 50. Since 50!/p is an integer, it must have a factorization, so 50!/p * p = 50 * 49 * 48 * … , but since p is relatively prime to each of 50, 49, 48…, that is not possible, therefore no other primes divide 50!, so it is only 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.
19. **A.** 6 and 24 are the two smallest perfect numbers because 1+2+3 = 6 and 1+2+4+7+14=28. The number is the sum of all of its factors less than the number.
20. **C** $6! = 2^4 * 3^2 * 5$, so x contains 0, 1 or 2 2's and 0 or 1 3's, so from that alone there are 6 possible values of x.
21. **D.** Let $a = p_1^{e_1} * p_2^{e_2} * ...$ and $b = q_1^{f_1} * q_2^{f_2} * ...$ for prime p's and q's and positive integer e's and f's, so $ab = (p_1^{e_1} * p_2^{e_2} * .)(q_1^{f_1} * q_2^{f_2} * ...)$

    a: $f(a) = (1+e_1)(1+e_2)...$, $f(b) = (1+f_1)(1+f_2)...$,
    $f(ab) = (1+e_1)(1+e_2)...(1+f_1)(1+f_2)...$

    b: Same argument as part a except with the sum of factors formula.

    c: f(a) will include all of the factors that a and 162 have in common, as f(b) will be for b, but since a and b are coprime, they share no factors so nothing is double counted in multiplying the two.

    d. Does not hold, i.e. a = 1, b= 162. $f(a) = f(b) = f(ab) = 162$
22. **D.** First of all, it is trivially true that they are all divisible by 1. Then, since $x^2 - 1 = (x-1)(x+1)$, $(x-1) \mid (x^2 - 1)$ and since x and $(x-1)$ are consecutive, at least one must be divisible by 2. Similar argument for 3: $x-1, x, x+1$. Finally, since $x(x^2-1)(x^2+1) = x^5 - x$ and $x \equiv x^5 \bmod 5$, that product must be divisible by 5, and since 5 is prime, at least one of the 3 terms must be divisible by 5. So there are 4 values of k: 1, 2, 3, 5.
23. **B** $5xy + 8z \equiv 4 \bmod 18 \rightarrow 7(5xy + 8z) \equiv (7*4) \bmod 18$, so it follows that $-xy + 2z \equiv 10 \bmod 18 \equiv 1 \bmod 9$
24. **E (6).** Numbers that have an odd number of distinct factors are squares (since all of their factors have even exponents, which is the only way to the product of one more than each could be odd). The two digit squares are $4^2 ... 9^2$, so 6 in all.
25. **C.** The sum of the first n cubes is $\frac{n^2(n+1)^2}{4}$, so in order for that to be divisible by 8, there must be at least 5 powers of 2 in the numerator, but since it's a square, the square root must contain at least 3, so the two smallest values are n = 7, 8.
26. **A.** $f(300) \equiv 600 \bmod 17 = 5$
27. **B.** Writing out the first few terms of that sequence mod4 gives $1,1,2,3,1,0,1,1,...$ which means it has a period of 6 and is divisible by 4 once in each period, so 20 times in the first 120.

28. **B.** For a number to be relatively prime to both 24 and 27, it must not be a multiple of 2 or 3. This means there are $108(\frac{1}{2})(\frac{2}{3}) = 36$ of them.
29. **C.** Any multiple of 3 can be expressed using the 3x alone and 12y will not be able to create anything the 3x cannot, so it is just the multiples of 3 and there are 66 of those less than 100.
30. **D.** Carl Friedrich Gauss.