2009 Mu School Bowl

Solutions

1. $f(x) = 2x^3 - 9x^2 + 12x + 1; f'(x) = 6x^2 - 18x + 12 \to \boldsymbol{A} = m = 54 + 54 + 12 =$ **<u>120</u>**

   $m = 24 - 36 + 12 = 0$; $\boldsymbol{B} = 16 - 36 + 24 + 1 =$ **<u>5</u>**

   $f''(x) = 0 = 12x - 18 \to x = 3/2$; $\boldsymbol{C} =$ **<u>11/2</u>**

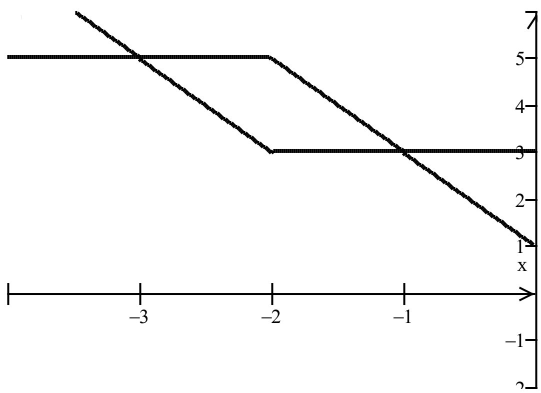
   $\int_0^3 (2x^3 - 9x^2 + 12x + 1)dx = \frac{1}{2}x^4 - 3x^3 + 6x^2 + x]_0^3 = \frac{81}{2} - 81 + 54 + 3 - 0 =$ **16.5** $= \boldsymbol{D}$

   $\sqrt{120 + 5 + 5.5 + 16.5} = \sqrt{147} = 7\sqrt{3}$

2. $A = \det\left[\begin{bmatrix} -2 & 5 & 1 \\ 3 & 7 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ 4 & 0 \\ 1 & 5 \end{bmatrix}\right]$; $\begin{bmatrix} -2 & 5 & 1 \\ 3 & 7 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 17 & 11 \\ 34 & -9 \end{bmatrix} \to A = \det\begin{bmatrix} 17 & 11 \\ 34 & -9 \end{bmatrix} = -527$

   $B = f(2) = 7; f(7) = 15; f(15) = 151$

   $C = 2\ \text{u}^2$



   $\boldsymbol{D} = Tan\left(Cos^{-1}\left(\frac{6}{7}\right)\right) = \frac{\sqrt{13}}{6}$

   $-527 - 151 + 2 + 13\left(\frac{\sqrt{13}}{6}\right)^2 = -\frac{24167}{36}$ or $-671\ {}^{11}/_{36}$

3. $\frac{A}{x} + \frac{B}{x-1} = \frac{3}{x(x-1)} \to Ax - A + Bx = 3; A + B = 0$ and $A = 3, B = -3 \to \frac{A}{x} + \frac{B}{x-1} = -\frac{3}{x} + \frac{3}{x-1}$

   $\boldsymbol{A} = \sum_2^{100} \frac{3}{(x-1)} - \frac{3}{x} = \frac{3}{1} - \frac{3}{2} + \frac{3}{2} - 1 + \frac{3}{4} - \frac{3}{4} + \ldots + \frac{3}{98} - \frac{3}{99} + \frac{3}{99} - \frac{3}{100} = \frac{297}{100}$.

   $\boldsymbol{B}$ = Zeros = 0

   $\boldsymbol{C} = \ln 2 + \frac{3}{8}$; $f'(x) = \frac{1}{x} - \frac{1}{4}x \to (f'(x))^2 = \frac{1}{x^2} - \frac{1}{2} + \frac{1}{16}x^2$; $\int_1^2 \sqrt{\left(\frac{1}{x^2} + \frac{1}{2} + \frac{1}{16}x^2\right)}\, dx = \int_1^2 \left(\frac{1}{x} + \frac{x}{4}\right)dx$

   $= \ln 2 + \frac{3}{8}$.

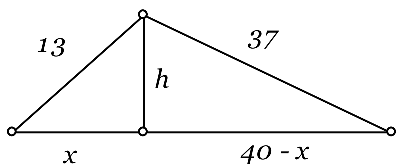
   $2x - y^2 - 2xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0 \to (3y^2 - 2xy)\frac{dy}{dx} = y^2 - 2x \to \frac{dy}{dx} = \frac{y^2 - 2x}{3y^2 - 2xy}$; $(-1, 2) = \frac{6}{16} = \frac{3}{8} = \boldsymbol{D}$

   $100\boldsymbol{A} - 4(\boldsymbol{B} + \boldsymbol{C} + \boldsymbol{D}) \to 297 - 4\left(0 + \frac{3}{8} + \ln 2 + \frac{3}{8}\right) = \boldsymbol{297 - 3 - 4\ln 2}$ or $\boldsymbol{294 - \ln 16}$.

4. $3x\boldsymbol{i} + 3y\boldsymbol{i} - 3\boldsymbol{i} + 2x = 5 + 7\boldsymbol{i} \rightarrow 3x + 3y = 10, 2x = 5 \rightarrow x = 2.5, y = {}^5/_6 \rightarrow x + y = {}^{10}/_3 = \boldsymbol{A}$

$$\sqrt{\frac{a}{b}\sqrt{\frac{b}{a}\sqrt{\frac{a}{b}}}} = \sqrt{\frac{a}{b}\sqrt{\frac{b}{a}\cdot\frac{a^{1/2}}{b^{1/2}}}} = \sqrt{\frac{a}{b}\sqrt{\frac{b^{1/2}}{a^{1/2}}}} = \sqrt{\frac{a}{b}\cdot\frac{b^{1/4}}{a^{1/4}}} = \sqrt{\frac{a^{3/4}}{b^{3/4}}}\cdot = \left(\frac{a}{b}\right)^{3/8}, k = {}^3/_8 = \boldsymbol{B}$$

$4^{\log_2 x} + x^2 = 8 \rightarrow 2^{\log_2 x^2} + x^2 = 8;\ 2x^2 = 8.\ x = \pm 2 \rightarrow \boldsymbol{x = 2 = C}$



$169 - x^2 = 1369 - (40 - x)^2;\ 400 = 80x; x = 5$, and $h = 12 = \boldsymbol{D}$

ABCD = $({}^{10}/_3)({}^3/_8)(2)(12) = \mathbf{30}$.

5. $y' = 25x^4 - 12x^2 - 2; y'' = 100x^3 - 24x;\ 4x(25x^2 - 6) = 0; x = 0, \pm\frac{\sqrt{6}}{5} \rightarrow\ \boldsymbol{A} = \frac{\sqrt{6}}{5}$

$\boldsymbol{B} = \mathbf{3}$; 1 possible positive zero and 0 possible negative zeros; $y'(-1) > 0, y'(0) < 0, y'(1) > 0 \rightarrow$ 2 local extrema.

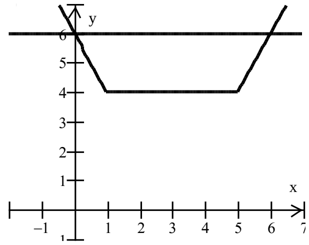
$\boldsymbol{C} = \int_0^1(-x - 4 - 5x^5 + 4x^3 + 2x + 4)dx = -\frac{5x^6}{6} + x^4 + \frac{x^2}{2}\Big]_0^1 = -\frac{5}{6} + 1 + \frac{1}{2} = {}^2/_3$

$\boldsymbol{D} = \lim_{x\to\frac{\sqrt{6}}{5}} \frac{100x^3 - 24x}{5x - \sqrt{6}} = \lim_{x\to\frac{\sqrt{6}}{5}} 4x(5x + \sqrt{6}) = \frac{4\sqrt{6}}{5}\cdot 2\sqrt{6} = {}^{48}/_5$

$\boldsymbol{A}^2 + \boldsymbol{BC} - \boldsymbol{D} = \frac{6}{25} + 3(2/3) - 48/5 = \left(-\frac{\mathbf{184}}{\mathbf{25}}\right)$

6. $\boldsymbol{A} = 6\pi$, since $\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$

$\boldsymbol{B} = 5$ Pairs: (5, 5),(4, 5),(3, 5),(2, 5),(1, 5) $y > |x - 1| + |x - 5|$ and $y < 6$.



$\boldsymbol{C} = \sqrt{5}$; $4(x + 3)^2 - (y + 5)^2 = 9 \rightarrow \frac{(x+3)^2}{\frac{9}{4}} - \frac{(y+5)^2}{9} = 1; \varepsilon = \frac{\sqrt{45/9}}{3/2} = \sqrt{5} = \boldsymbol{C}$

$\boldsymbol{D} = 12; \frac{(x+2)^2}{36} - \frac{(y-4)^2}{9} = 1 \rightarrow a = 6$; transverse axis = 12.

$\boldsymbol{AB} + \boldsymbol{CD} = 30\pi + 12\sqrt{5}$.

7. $\boldsymbol{A} = 5; x = \frac{87-3y}{5} \rightarrow$ (15, 4), (12, 9),(9, 14),(6, 19),(3, 24).

$\boldsymbol{B} = -448$; 6th term = $\binom{8}{5} = \frac{8!}{5!3!}(2x)^3(-y)^5 = -448x^3y^5$.

$\boldsymbol{C} = 2;\ y^2 = x - 1 + \frac{6}{x} \rightarrow x = 1,2,3,6$ and $y^2 = 6, 4, 4, 6 \rightarrow (2, 4)$ and $(3, 4)$.

$\boldsymbol{D} = \frac{1}{3};\ \left(\sqrt[3]{3\sqrt{3}}\right)^{-2} = \left(3^{\frac{3}{2}}\right)^{-\frac{2}{3}} = 3^{-1}$.

$\frac{\boldsymbol{B}}{\boldsymbol{ACD}} = \frac{-448}{5\cdot 2(1/3)} = \mathbf{-134.4}$ **or** $-\frac{672}{5}$

8. $\sum_2^{2008} log_2 \frac{n}{n+1} = a + blog_2\, c = log_2 \frac{2}{3} + log_2 \frac{3}{4} + log_2 \frac{4}{5} + \cdots + log_2 \frac{2007}{2008} + log_2 \frac{2008}{2009} = log_2 \frac{2}{2009} =$ $1 + -1\ log_2\ 2009 \rightarrow \boldsymbol{A} = 2009$.

$ln(3x - 1) + ln(x + 3) = 2\ ln\ 5;\ ln(3x - 1)(x + 3) = ln\ 25;\ 3x^2 + 8x - 28 = 0 \rightarrow (3x + 14)(x - 2) = 0$

$\boldsymbol{B} = 2$

$2log_n(a\sqrt{b}) - log_n b^2 c = 2log_n a + log_n b - 2log_n b - log_n c \rightarrow 4 + 3 - 6 - 5 = -4$. $\boldsymbol{C} = -4$.

$\frac{2\log x}{2\log y} + \frac{\log x}{2\log y} = 6;\ 3log\ x = 12\ log\ y \rightarrow x = y^4;\ k = 4$. $\boldsymbol{D} = 4$

$\boldsymbol{2A}/\boldsymbol{B} + \boldsymbol{CD} = 2009 - 16 = 1993 \rightarrow$ **22**

9. $f'(x) = x(1 - x^2)^{1/2} \rightarrow f(x) = -\frac{1}{3}(1 - x^2)^{3/2} + C;\ f(x) = -\frac{1}{3}(1 - x^2)^{3/2} + \frac{10}{3};\ f\left(\frac{1}{2}\right) = \frac{-3\sqrt{3}+80}{24} = \boldsymbol{A}$.

$lim_{x\rightarrow\infty} \frac{\ln(x^2+1)}{\ln x} = \lim_{x\rightarrow\infty} \frac{2x\left(\frac{1}{x^2+1}\right)}{\frac{1}{x}} \rightarrow \lim_{x\rightarrow\infty} \frac{2x^2}{x^2+1} = \lim_{x\rightarrow\infty} \frac{2}{1+\frac{1}{x^2}} = 2;\ \boldsymbol{B} = 2$

$2ysin\ 2x \frac{dy}{dx} + 2y^2cos\ 2x = 2\frac{dy}{dx} \rightarrow \frac{dy}{dx} = \frac{-2y^2 \cos 2x}{2ysin\ 2x - 2};\ \boldsymbol{C} = \frac{-8(0)}{-4(-1)-2} = 0$

$A = \int_{-1}^{1}(3x^2 + 4)dx \rightarrow x^3 + 4x\Big|_{-1}^{1} = 5 + 5 = 10$. $\boldsymbol{D} = 10$.

$\boldsymbol{(A + C)(B + D)} = \left(\frac{-3\sqrt{3}+80}{24} + 0\right)12 = \frac{\mathbf{-3\sqrt{3}+80}}{\mathbf{2}}$

10. Let $u = \log_9 x \rightarrow u + 1/u = 5/2$; $2u^2 - 5u + 2 = 0$; $(2u - 1)(u - 2) = 0 \rightarrow \log_9 x = ½$, $x = 3$; $\log_9 x = 2$, $x = 81$. $\boldsymbol{A} = 3$.

$a(1 + r + r^2) = 21$, $\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} = {}^7\!/_{12}$; $\frac{r^2+r+1}{ar^2} = {}^7\!/_{12} \rightarrow \frac{21}{a^2r^2} = {}^7\!/_{12}$; $a^2r^2 = 36$, $a = 2$, $r = 3$
3, 6, 12. $\boldsymbol{B} = 3$.

$2l = 12$, $l = 6$, diagonal of $R_1 = 2\sqrt{10}$. $\frac{x}{6} = \frac{15}{2\sqrt{10}}$; $x = \frac{45}{\sqrt{10}}$. $\frac{x}{2} = \frac{15}{2\sqrt{10}}$, $x = \frac{15}{\sqrt{10}}$; $A = \frac{45}{\sqrt{10}} \cdot \frac{15}{\sqrt{10}} = \frac{135}{2}$

$\sqrt{20 + \sqrt{384}} = \sqrt{a} + \sqrt{b} \rightarrow a + b = 30$, $ab = 2\sqrt{96}$. $\sqrt{8} + \sqrt{12} = 8 + 12 = 20$. $\boldsymbol{D} = 20$.

$$\frac{\boldsymbol{C^2}}{A^B D} = \frac{135}{2} \cdot \frac{135}{2} \cdot \frac{1}{27 \cdot 20} = \boldsymbol{\frac{135}{16}}$$