

2009 Discrete Math (Open)

1. If p and q are logical propositions, then the statement $A \vee 1 = 1$ is an example of
 - A. a contradiction
 - B. Distributive Law
 - C. Demorgan's Law
 - D. Domination Law
 - E. NOTA

2. Order the following functions in growth order from slowest to fastest.
I. $y = x^2$ II. $y = \ln x$ III. $y = x!$ IV. $y = e^x$
 - A. I, II, III, IV
 - B. II, I, III, IV
 - C. II, I, IV, III
 - D. I, IV, II, III
 - E. NOTA

3. In the integers modulo 10, which of the following equations is unsolvable?
 - A. $3x = 4$
 - B. $5x = 4$
 - C. $7x = 4$
 - D. $9x = 4$
 - E. NOTA

4.
$$\sum_{n=1}^4 \sum_{j=1}^n (j+1)$$
 - A. 20
 - B. 30
 - C. 50
 - D. 80
 - E. NOTA

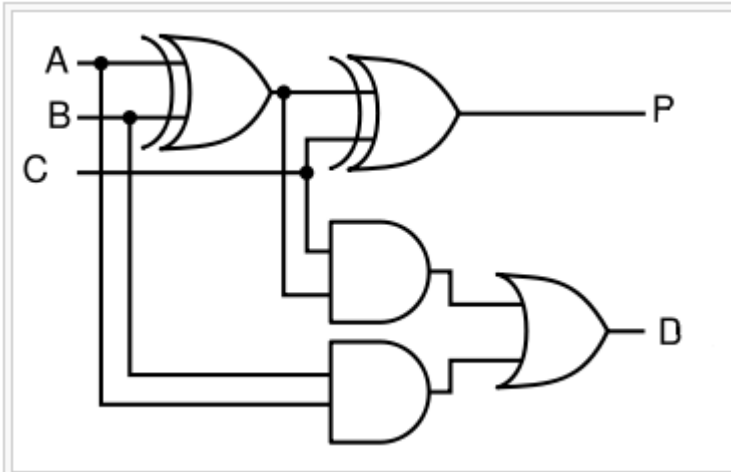
5. What is the sum of the digits in the number of distinguishable permutations of MISSISSIPPI?
 - A. 18
 - B. 19
 - C. 20
 - D. 21
 - E. NOTA

6. How many distinct factors does 2016 have?
 - A. 8
 - B. 10
 - C. 36
 - D. 72
 - E. NOTA

7. Find the sum of all the entries in the multiplicative reciprocal of the 8×8 identity matrix.
 - A. -8
 - B. $\frac{1}{8}$
 - C. 0
 - D. 8
 - E. NOTA

8. Consider the positive integers modulo 7. What is the multiplicative reciprocal of 2?
 - A. $\frac{1}{2}$
 - B. -2
 - C. 3
 - D. 4
 - E. NOTA

9.



Which of the following best describes the purpose of the circuit?

- | | | |
|-------------------|------------------------|-----------------|
| A. Addition | B. Addition with carry | C. Exclusive-OR |
| D. Multiplication | E. NOTA | |

10. Which of the following propositions are tautologies?

- I) $p \rightarrow q \Leftrightarrow q \rightarrow p$
- II) $p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$
- III) $p \rightarrow q \Leftrightarrow \neg p \vee q$

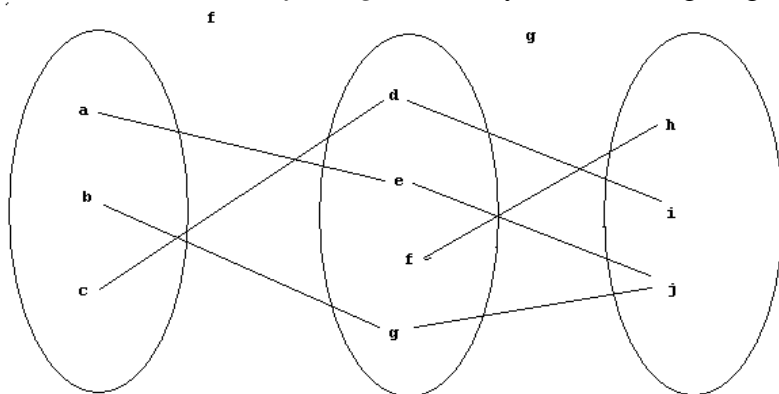
- | | | |
|------------------|------------|--------------------|
| A. I only | B. II only | C. II and III only |
| D. I, II and III | E. NOTA | |

11. 1) Even Integers
 2) Odd Integers
 3) Positive Integers
 4) Integers
 5) Rational Numbers

Each set has an associated CARDINALITY. How many distinct cardinalities are represented by these sets?

- | | | |
|----------------|---------|------|
| A. 1 | B. 2 | C. 3 |
| D. more than 3 | E. NOTA | |

12 Consider the relations f and g defined by the following diagram.



Which of the following are true about the composition of $f \circ g$ on the domain $\{a, b, c\}$

- I) Function
- II) One to One
- III) Onto

- A. I only
- B. II only
- C. III only
- D. I and II only
- E. NOTA

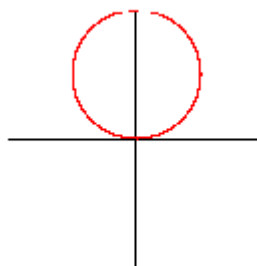
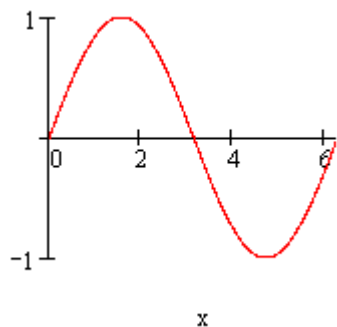
13 Consider the equation $y = \sin(x)$ on the domain $[0, 2\pi]$.

Graph (1) is y graphed with Cartesian Coordinates.

Graph (2) is y graphed with Polar Coordinates.

What can you conclude about y from the two graphs?

- (1) Cartesian
- (2) Polar



- A. y is **NOT** a function because it fails the vertical line test on graph (2).
- B. y is **NOT** a function because it fails the horizontal line test on graph (2).
- C. y is **NOT** a function because it fails both the horizontal and vertical line tests on graph (2).
- D. It can not be determined if y is a function.

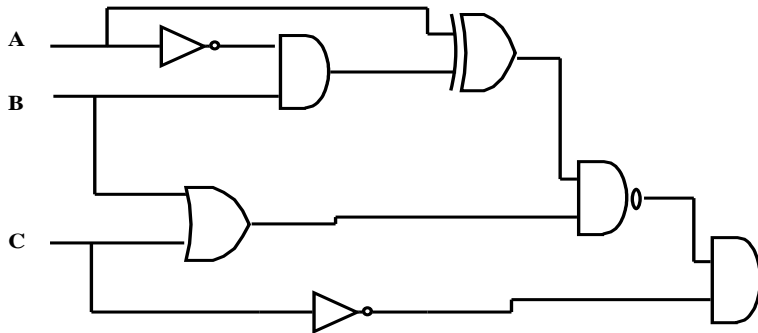
E. NOTA

14 The relation R is defined as follows:

A is related to B , $A R B$, if $A^2 + B^2 = 1$. Which of the following apply to R .

- I. Reflexive
- II. Symmetric
- III. Transitive

- A. I only
- B. II only
- C. I, II
- D. I, II, and III
- E. NOTA



15 Which of the following ordered triples make the circuit TRUE. (Note: 1=true, 0=false, *=either/both)

- A. (1,1,1)
- B. (0,1,*)
- C. (*,0,0)
- D. (*,*,0)
- E. NOTA

16. How many paths of length 2 are in the graph represented by the following adjacency matrix?

	A	B	C	D
A	1	1	0	0
B	0	0	1	0
C	0	1	0	1
D	0	0	1	0

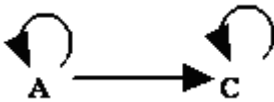
- A. 6
- B. 7
- C. 8
- D. 9
- E. NOTA

17 Which of the following changes to the graph and the associated adjacency matrix will create the fewest paths of length 2?

	A	B	C	D
A	1	1	0	0
B	0	0	1	0
C	0	1	0	1
D	0	0	1	0

- A.
- B.
- C.

D.



E. NOTA

18 Convert $0.B86_{16}$ to a base 10 proper fraction reduced to lowest terms.

A. $\frac{1}{2}$

B. $\frac{1}{40}$

C. $\frac{1251}{2048}$

D. $\frac{1475}{2048}$

E. NOTA

19. Find $f(12,3)$

$$f(x,y) = \begin{cases} f(x/2, y-1) + 3x & |x| \text{ is even } |y| \text{ is odd} \\ f(y-2, x+1) + x & |x| \text{ is even } |y| \text{ is even} \\ x^2 - y^2 & |x| \text{ is odd } |y| \text{ is even} \\ f(y+1, x-1) - 2y & |x| \text{ is odd } |y| \text{ is odd} \end{cases}$$

A. 47

B. 11

C. 5

D. -3

E. NOTA

20. Definitions:

$a!b$ = Greatest Common Factor (GCF) of a and b .

$a\#b$ = Least Common Multiple (LCM) of a and b .

$a^b = a^b$

Evaluate the following prefix expression:

$$\wedge / * ! 12 \ 18 \ # \ 2 \ 3 \ ^ ! 6 \ 8 \ 2 ! 9 \ 15$$

A. 567

B. 64

C. 125

D. 729

E. NOTA

21 If A is the non-augmented matrix associated with the following system of equations, which of the following matrices, B , would cause $C = B \times A$ to **NOT** have a zero in position $C_{(2,3)}$?

$$2x + 3y + 2z = 4$$

$$3x + 5y + 3z = 5$$

$$5x + 2y + 7z = 15$$

A. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -7 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 7 & -3 \\ 0 & 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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D.
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
 E. NOTA

22 Let A and B be Boolean variables. Let ADDITION represent the Boolean operation OR, MULTIPLICATION represent the Boolean operation AND, and an overbar represent NOT. Which of the following Boolean statements is false?

- A. $A+B = B+A$ B. $A(B+C) = AB+AC$ C. $A(B+C) = AB+AC$
 D. $\overline{A+B} = \overline{A} \cdot \overline{B}$ E. NOTA

23 Two events, A and B , have non-zero probabilities. The two events are mutually exclusive. Which of the following is false.

- A. A and B can be independent B. $P(A \cap B) = 0$ C. $P(A) + P(B) = P(A \cup B)$
 D. $P(A) + P(B) \leq 2$ E. NOTA

24 A bag contains one white marble and two red marbles. A marble is drawn at random. If the marble is white then it is put back in the bag along with another white marble. If the marble is red then it is put back in the bag with two extra red marbles. Find the probability that the first marble drawn is red, given that the second marble was red.

- A. $\frac{7}{10}$ B. $\frac{8}{15}$ C. $\frac{16}{21}$
 D. $\frac{4}{5}$ E. NOTA

25 In Poker, a flush consists of five cards of the same suit. The first four cards in a five-card poker hand are hearts. What is the probability of getting a flush with the next card?

- A. $\frac{9}{48}$ B. $\frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}$ C. $\frac{12 \cdot 11 \cdot 10 \cdot 9}{51 \cdot 50 \cdot 49 \cdot 48}$
 D. $\frac{1}{13}$ E. NOTA

26 Two cards are drawn from a standard deck of cards. What is the probability that the second card drawn is an Ace?

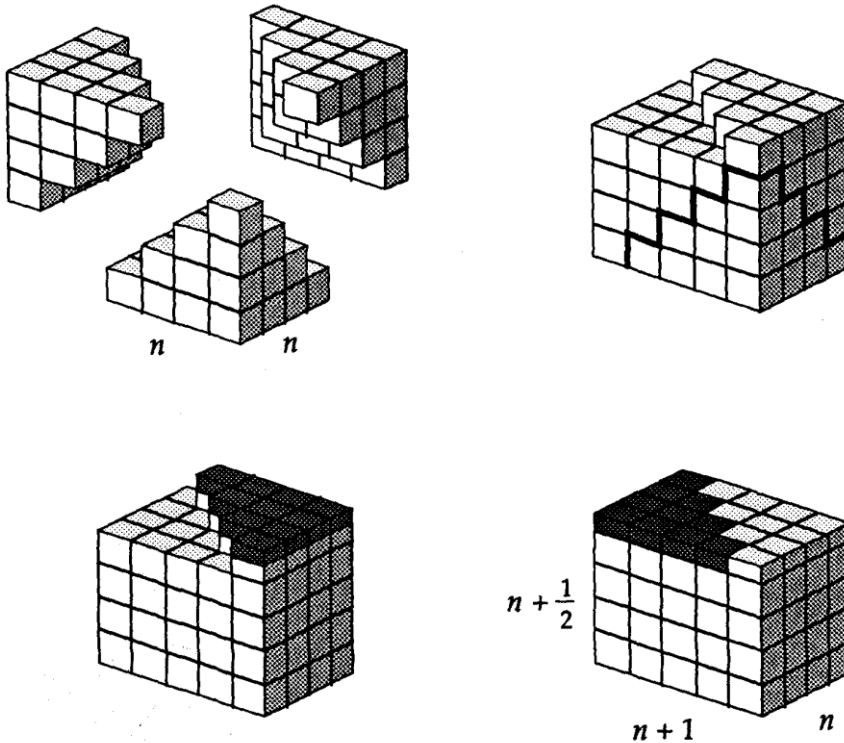
- A. $\frac{1}{17}$ B. $\frac{3}{16}$ C. $\frac{1}{13}$
 D. $\frac{67}{272}$ E. NOTA

- 27 In positional notation, numerals are created by the juxtaposition of allowable digits. Each digit is multiplied by the "column value" and then summed to get the complete value. In a new number system more complex "digits" are allowed. In this new system, the only change is that digits are completely defined by enclosing any base-10 expression in parenthesis. What is the standard base-10 value of the following base-4 number?

$$\left(\frac{1}{2}\right)(-1)(5)(3)$$

- A. -7.5
B. 7.5
C. 39
D. 71
E. NOTA
- 28 On a game show, there are three closed doors concealing, respectively, a new car and two goats. The contestant must first choose one of the doors. At this point, the host of the game show, opens a door to reveal one of the goats and gives the contestant the option of switching his choice to the other closed door. Which of the following statements is true. (Assuming that the contestant wants to win the car!)
- A. The probability of winning is $\frac{1}{2}$ whether the contestant switches doors or not.
B. The probability of winning is $\frac{1}{2}$ only if the contestant changes doors.
C. The probability of winning is $\frac{2}{3}$ if the contestant doesn't change doors.
D. The probability of winning is $\frac{2}{3}$ only if the contestant changes doors.
E. NOTA

29



Which of the following equations is best demonstrated by the diagram?

A. $\sum_{i=1}^n i = \frac{n(n+1)}{2}$

B. $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$

C. $\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$

D. $\sum_{i=1}^n i^4 = \frac{1}{3}n(n+1)\left(n + \frac{1}{2}\right)$

E. NOTA

30 What conclusion can be logically drawn from the following statements?

1. No kitten, that loves fish, is unteachable;
2. No kitten without a tail will play with a gorilla;
3. Kittens with whiskers always love fish;
4. No teachable kitten has green eyes;
5. No kittens have tails unless they have whiskers.

- A. Kittens with green eyes will play with a gorilla
- B. Gorillas will play with kittens with green eyes
- C. Kittens who love fish have tails.
- D. No kitten with green eyes will play with a gorilla
- E. NOTA