Alpha Ciphering 2009

Solutions:

0. Find the number of permutations of the in the word Mississippi.

$$\frac{11!}{4!4!2!} = 34650$$

1. One of the roots of $x^3 - 2x^2 - 5x + 6 = 0$ is 1. Find the other 2 roots.

Solution: $(x-1)(x^2-x-6) = 0 \rightarrow (x-1)(x-3)(x+2) = 0$ **Solution:** 3, -2

2. A square is inscribed in another square such that the vertices of the inscribed square divides the sides of the outside square in a ratio of 3:1. If the difference in the area of the 2 squares is 16 sq. units find the area of the larger square.

Solution:

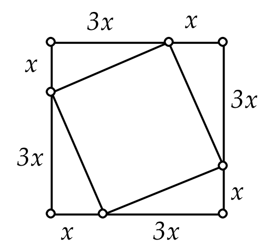
$s^2 = 10x^2$, Area of larger square $= 16x^2$

$A^2 - s^2 = 16 = 16x^2 - 10x^2$

$6x^2 = 16$

$x^2 = \frac{8}{3} \rightarrow A = \frac{128}{3}$ or $42\frac{2}{3}$

Solution: $\frac{128}{3}$ or $42\frac{2}{3}$



3. Simplify: $\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{2009} - \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{2009}$

Solution:

$r = 1; \tan^{-1}\frac{\sqrt{3}}{3} = \frac{\pi}{3} \rightarrow 1^{2009}\left(\sin\frac{2009\pi}{3} + \cos\frac{2009\pi}{3}i\right) \rightarrow \left(\sin\frac{5\pi}{3} + \cos\frac{5\pi}{3}i\right) \rightarrow \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) - \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \sqrt{3}$

4. Find the sum of all real x such that $\sqrt{\frac{x+4}{x-1}} + \sqrt{\frac{x-1}{x+4}} = \frac{5}{2}$.

Solution: $x + 4 = a, x - 1 = b \rightarrow \sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = \frac{5}{2} \rightarrow \frac{a}{b} + 2 + \frac{b}{a} = \frac{25}{4} \rightarrow a^2 + 2ab + b^2 = \frac{25}{4}ab$

$4a^2 + 8ab + 4b^2 = 25ab \rightarrow 4a^2 - 17ab + 4b^2 \rightarrow (4a-b)(a-4b) \rightarrow a = \frac{b}{4}, a = 4b$

$x + 4 = \frac{x-1}{4} \rightarrow 4x + 16 = x - 1 \rightarrow x = -\frac{17}{3}$; $x + 4 = 4x - 4 \rightarrow x = \frac{8}{3} \rightarrow$ Sum = -3

5. $\log x = 2; \log y = -3; \log z = 5$. Find the numerical equivalent of $\log\frac{x^2\sqrt{xyz}}{y^3\sqrt[3]{z}}$.

Solution: $2\log x + \frac{1}{2}(\log x + \log y + \log z) - 3\log y - \frac{1}{3}\log z \rightarrow 4 + \frac{1}{2}(2 - 3 + 5) + 9 - \frac{5}{3} \rightarrow \frac{40}{3}$

6. Solve: $\tan 2\theta + 2\sin\theta = 0$ where $0 \le \theta < 2\pi$.

   Solution: $\frac{\sin 2\theta}{\cos 2\theta} + 2\sin\theta = 0 \rightarrow 2\sin\theta\cos\theta + 2\sin\theta(2\cos^2\theta - 1) = 0 \rightarrow 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$.

7. Solve for X; $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} X = 2\begin{bmatrix} -1 & 4 & 5 \\ 2 & 3 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 2 \end{bmatrix}$

   Solution: $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} X = 2\begin{bmatrix} 9 & 9 \\ 1 & 2 \end{bmatrix} \rightarrow -\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 18 & 18 \\ 2 & 4 \end{bmatrix} \rightarrow X = \begin{bmatrix} -66 & -60 \\ 50 & 48 \end{bmatrix}$

8. Solve the system: $\begin{cases} 5xy + 13y^2 + 36 = 0 \\ xy + 7y^2 = 6 \end{cases}$

   Solution: $5(6 - 7y^2) + 13y^2 + 36 = 0 \rightarrow 66 = 22y^2 \rightarrow y = \pm\sqrt{3}; x\pm\sqrt{3} + 21 = 6 \rightarrow x = \mp 5\sqrt{3}$
   $(-5\sqrt{3}, \sqrt{3})$ and $(5\sqrt{3}, -\sqrt{3})$

9. When a certain polynomial is divided by $x - 2$, the remainder is 2. When the polynomial is divided by $x + 2$, the remainder is -2. What is the remainder when the polynomial is divided by $x^2 - 4$?

   Solution: Assume a quadratic of form $ax^2 + bx + c = f(x)$; $f(2) = 2 = 4a + 2b + c$
   $f(-2) = -2 = 4a - 2b + c$
   $4a + c = 0 \rightarrow$ Let $c = -4$, then $a = 1$ and $b = 1$ and polynomial is $x^2 + x - 4$, and the remainder is x.

10. Let $f(0) = -2$, $f(1) = 3$ and $f(n + 1) = 2f(n) - f(n - 1)$ for $n > 1$. Find $f(2009)$

    Solution:

n	2	3	4	5	6	7	8
$F(n + 1)$	8	13	18	23	28	33	38

$f(n + 1) = 5n - 2 \rightarrow f(2009) = 10043$