Solutions:

0. $\frac{7!}{4!} = 210$

1. If $\frac{dy}{dx} = 3x^2 - \sqrt{x+1} + 2$ and $f(3) = -2$, find $f(x)$.

   $y = x^3 - \frac{2}{3}(x+1)^{\frac{3}{2}} + 2x + c \rightarrow 3^3 - \frac{2}{3}(4)^{\frac{3}{2}} + 6 + c = -2 \rightarrow 27 - \frac{16}{3} + 6 + c = -2 \rightarrow c = -35 + \frac{16}{3}$;

   $\mathbf{f(x) = x^3 - \frac{2}{3}(x+1)^{\frac{3}{2}} + 2x - \frac{89}{3}}$

2. If the roots of the polynomial $ax^3 + bx^2 + cx + d = 0$ are $-2, 1 \pm \sqrt{5}$, find the sum of $a + b + c + d$

   $r_1r_2r_3 = 8$, $r_1 + r_2 + r_3 = 0$, $r_1r_2 + r_1r_3 + r_2r_3 = -8 \rightarrow x^3 - 8x - 8 = 0 \rightarrow \mathbf{a + b + c + d = -15}$

3. Find: $\lim_{t\rightarrow 0} \frac{4t^2+3t\sin t}{t^2}$.

   $$\lim_{t\rightarrow 0} \frac{\frac{4t^2}{t^2}+\frac{3t}{t^2}\sin t}{1} \rightarrow \lim_{t\rightarrow 0} \frac{4+\frac{3}{t}\sin t}{1} \rightarrow 7$$

4. Find all ordered pairs (x, y) of real numbers for which $x^2 + xy + x = 14$ and $y^2 + xy + y = 28$.

   $2x^2 + 2xy + 2x - y^2 - xy - y = 0 \rightarrow 2x^2 + x(y + 2) - (y^2 - y) = 0 \rightarrow x = \frac{-(y+2)\pm\sqrt{9y^2+12y+4}}{4}$

   $x = \frac{-y-2 \pm 3y+2}{4}$; $x = \frac{1}{2}y$ and $x = -y - 1$; $x^2 + 2x^2 + x - 14 = 0 \rightarrow (3x + 7)(x - 2) \rightarrow x = 2, -\frac{7}{3}$

   $\mathbf{(2, 4), \left(-\frac{7}{3}, -\frac{14}{3}\right)}$; $x^2 + x(-x - 1) + x = 14 \rightarrow 0 \neq 14$, $x = -y - 1$ does not work.

5. A sequence a_n is defined as follows: $a_1 = 2$, $a_n = 3a_{n-1} + 2$ for $n > 1$. Find the term a_{2009}.

n	1	2	3	4	5
a_n	2	8	26	80	242

   $3^1 - 1, 3^2 - 1, 3^3 - 1, 3^4 - 1, 3^5 - 1, \ldots, \mathbf{3^{2009} - 1}$.

6. Evaluate: $\int_1^4 \frac{x+16}{x^2+2x-8}dx$.

   $\frac{A}{x+4} + \frac{B}{x-2} = \frac{x+16}{x^2+2x-8} \rightarrow \frac{Ax-2A+Bx+4B}{(x+4)(x-2)} = \frac{x+16}{x^2+2x-8} \rightarrow A + B = 1; -2A + 4B = 16$

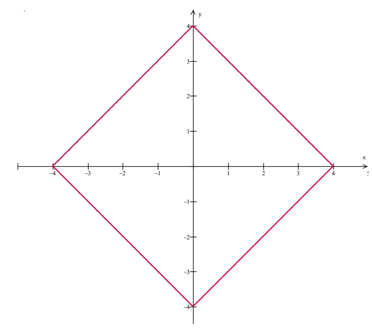
   $6B = 18 \rightarrow B = 3, A = -2 \rightarrow \int_1^4 \frac{3}{x-2}dx - \int_1^4 \frac{2}{x+4}dx \rightarrow 3\ln(x-2) - 2\ln(x+4) \Big]_1^4 = \ln\frac{|x-2|^3}{(x+4)^2}\Big]_1^4$

   $\ln\frac{1}{8} - \ln\frac{1}{25} \rightarrow \mathbf{\ln\frac{25}{8}}$.

7. Find the number of lattice points defined by the region $|x| + |y| < 4$.
(A lattice point in a rectangular coordinate plane is a point both whose coordinates are integers.)

(1, 1), (1,2), (2, 1) times 4 = 12
(-3, 0), (-2, 0), (-1, 0), (1, 0), (2, 0), (3, 0) = 6
(0, -3), (0, -2), (0, -1), (0, 1), (0, 2), (0, 3) = 6
(0, 0) = 1

Total: **25**



8. If the domain for x is complex numbers, find the solution set of $9x^4 + 20x^2 + 16 = 0$. Express each element of the solution set in the form $a + b\boldsymbol{i}$, where a and b are real numbers.

$9x^4 + 24x^2 + 16 - 4x^2 = 0 \rightarrow (3x^2 + 4)^2 - 4x^2 = 0 \rightarrow (3x^2 - 2x + 4)(3x^2 + 2x + 4) = 0$

$x = \dfrac{2 \pm \sqrt{4-48}}{6}$; $x = \dfrac{-2 \pm \sqrt{4-48}}{6} \rightarrow \boldsymbol{x = \frac{1}{3} \pm \frac{\sqrt{11}}{3} i;\ -\frac{1}{3} \pm \frac{\sqrt{11}}{3} i}$

9. In a simple code, each letter of the alphabet is assigned its numerical position in the alphabet. A one word message was received in this code, but was lost. All that the operator remembered was that the message had the form of x, $x + 7$, $x + 6$, $x + 5$, that the second letter was a vowel, and that the word was an English word. What was the one word message?

Possible vowels: *a, e, i, o, u*. Position in alphabet: 1, 5, 9, 15, 21 and only $x + 7$ can represent *i*, *o*, and *u*. If it is *i* then word would be BIHG. If it is *o* then word would be JONM and for *u* the word would be **NUTS**.

10. Find the equation of the tangent line to the graph $2x^3 - x^2y + y^3 - 1 = 0$ at point $\left(\frac{1}{2}, 1\right)$

$6x^2 - 2xy - x^2\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0 \rightarrow \frac{dy}{dx} = \frac{2xy - 6x^2}{3y^2 - x^2} \rightarrow m = -\frac{2}{11} \rightarrow y - 1 = -\frac{2}{11}\left(x - \frac{1}{2}\right) \rightarrow$

$\boldsymbol{2x + 11y = 12}$