1. **D** $2i(3-4i)-(1-i)(2+5i)=(6i+8)-(7+3i)=1+3i$
2. **D** We see that $\overline{wz}=\overline{(a+bi)(c+di)}=\overline{(ac-bd)+(ad+bc)i}=(ac-bd)-(ad+bc)i$.
3. **A** $(1-2i)^4=(1-4i-4)^2=(-3-4i)^2=(3+4i)^2=-7+24i$.
4. **C** Note that $\frac{1}{3}e^{-i\theta}\cdot 3e^{i\theta}=1$. In rectangular form, $\frac{1}{3}e^{-i\theta}=\frac{1}{3}(\cos(-i\theta)+i\sin(-i\theta))$. In vector form, we can rewrite this as $\left\langle\frac{\cos(\theta)}{3},-\frac{\sin(\theta)}{3}\right\rangle$.
5. **A** $\frac{3}{1-4i}=\frac{3}{1-4i}\cdot\frac{1+4i}{1+4i}=\frac{3}{17}+\frac{12}{17}i$.
6. **E** By inspection, we see $z=0$ is a real zero of $f(z)=z^5+(5-2i)z^4-(3+10i)z^3+(6i-15)z^2+30iz$, leaving us with $z^4+(5-2i)z^3-(3+10i)z^2+(6i-15)z^1+30i$. The rational roots theorem holds, so we find, through trial and error, that $z=-5$ is also a real root, leaving us with $z^3-i2z^2-3z+6i=(z^2-3)(z-2i)$. The factor z^2-3 yields two more real zeros, so the total is four.
7. **D** Note that $|\overline{z}|=2\sqrt{5}$, so $(a+bi)(a-bi)=20$, and we have$=\frac{25}{z}-\frac{15}{\overline{z}}=\frac{25}{a+bi}-\frac{15}{a-bi}=\frac{25(a-bi)-15(a+bi)}{20}=\frac{1}{2}a-2bi=1-8i$, so $a=2$ and $b=4$.
8. **B** $\sqrt[\pi]{(-1)^{-i}}=(-1)^{-i/\pi}=(e^{\pi i})^{(-\frac{i}{\pi})}=e^1$
9. **A** Since we don't know which, if any, of the coefficients are real, there is no guarantee that complex roots come in conjugate pairs. Thus, the minimum possible number of real zeros is 0.
10. **D** The function has domain restrictions when the denominator equals zero; that is, when $|z|^2=1$. This includes all points with modulus one; or in other words, all points lying on the unit circle.
11. **D** Consider i^{7^k} Upon inspection, we see that when k is odd,$i^{7^k}=-i$, and when k is even, $i^{7^k}=i$. Thus, $i^{7^{127}}=-i$.
12. **A** Which of the following are true for $z\in\mathbb{C}$?

    I. **FALSE** $|\cos(z)|$ is not bounded, for $|\cos(z)|=|\cosh(iz)|$. Taking $z=-in$ with $n\in\mathbb{N}$, if $n\to\infty$ we have also that $|\cosh(n)|\to\infty$.

    II. **TRUE** - If $w=r_1e^{i\theta}$ and $z=r_2e^{i\psi}$, then $wz=r_1r_2e^{i(\theta+\psi)}$.

    III. **FALSE** - complex variables give us the tools evaluate non-real z by appealing to Euler's formula.
13. **B** $\arcsin(\cosh(i\frac{4\pi}{3}))=\arcsin(\cos(i(i\frac{4\pi}{3})))=\arcsin\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$, since the range of arcsin is $[-\pi/2,\pi/2]$.
14. **C** Let $\ln(-ei)=\ln(e)+\ln(-i)=1+\ln(e^{-\frac{\pi}{2}i})=1-\frac{\pi}{2}i$.
15. **A** The pigeon-hole principal says that if you have n pigeonholes, and more than n items to place in the holes, if you were to place one item in each hole, at least one hole would have more than one item. No complex variables are needed in this counting principal.
16. **C** $\cos(3i\ln(i))=\cos(\ln(i^{3i}))=\cos(\ln(e^{(\frac{\pi}{2}i)(3i)}))=\cos(-\frac{3\pi}{2})=0$.
17. **C** Set $z=\frac{1}{z}$ so that $z^2=1$. The only z that satisfy are the two values $z=\pm1$.
18. **E** Set $z=\frac{1}{\overline{z}}$ so that $z\overline{z}=1$; that is, $a^2+b^2=1$ - in other words, any $z=a+bi$ that lies on the unit circle. There are infinitely many of these.
19. **B** Note that all three of these points lie three units from $1+i$, which is thus the center. The modulus of $1+i$ is $\sqrt{2}$, which lies in $[1,2)$.

20. **C**

   I. $\mathbb{H}$ is commutative under addition **TRUE** - If we have $w = a + bi + cj + dk$ and $z = e + fi + gj + hk$, then $w + k = (a+e) + (b+f)i + (c+g)j + (d+h)k = (e+a) + (f+b)i + (g+c)j + (h+d)k = k + w$.

   II. $\mathbb{H}$ is commutative under multiplication **FALSE** - Counterexample: We already have that $jk = -kj$, so multiplication is not commutative.

   III. $\mathbb{H}$ is associative under addition **TRUE** - We omit the proof here, but the proof is very similar to the one for commutativity under addition.

21. **B** Note that the order of multiplication matters with Quaternions: $5i \cdot 2j \neq 2j \cdot 5i$. Now, $zw = (5 + 3j + 2k)(3i - k) = 15i - 5k + 9ji - 3jk + 6ki - 2k^2 = 15i - 5k - 9k - 3i + 6j + 2 = 2 + 12i + 6j - 14k$. Thus, $a + b + c + d = 2 + 12 + 6 + -14 = 6$.

22. **D** Sir William Hamilton was entranced with using the complex numbers to represent vectors in two dimensions, and searched in vain for a similar construction in three dimensions. While taking a stroll with his lady one day, the idea struck him that he actually needed to look to a *four* dimensional system, which became known as the Hamiltonian Quaternions. The i, j, k of the Quaternions became the unit vectors of 3D vectors, and the constants are ignored in normal vector analysis. The Hamiltonian Quaternions were a hugely important discovery (creation?), as they were the first example of a number system that was not commutative under multiplication.

23. **D** If $\Im(z) = 6$, $|\overline{z}| = 8$, and if $z = a + bi$, then $b = 6$ and $a^2 + b^2 = 8^2$. Thus $a = \pm 4\sqrt{7}$.

24. **B** Consider that $z + \overline{z} = (a + bi) + (a - bi) = 2a = 2$, so $a = 1$. Since b is free to take on any value, this must be a vertical line.

25. **C** $\left|\frac{3+2i}{1-i}\right| = \left|\frac{(3+2i)(1+i)}{2}\right| = \left|\frac{1+5i}{2}\right| = \sqrt{\frac{1+5i}{2} \cdot \frac{1-5i}{2}} = \frac{\sqrt{26}}{2}$. Thus $p = \sqrt{26}$.

26. **D** When $p = 0$, there are five possibilities: $q = 0, \pm 1, \pm 2$. When $p = \pm 1$, we can have $q = 0, \pm 1, \pm 2$, which constitute 10 possibilities. Finally, when $p = \pm 2$, we can have $q = 0, \pm 1$, which yields six possibilities. There are thus 21 such values of z that satisfy.

27. **C** Since $a_2 = -3i$ and $a_5 = 24$, we can write $a_5 = a_2 r^3 = 24$, so $r^3 = 8i$. Thus r could equal $-2i, \pm\sqrt{3} + i$. Taking $r = -\sqrt{3} + i$, we have $a_7 = 24(-\sqrt{3} + i)^2 = 48 - i48\sqrt{3}$.

28. **A** $|5 - 3i| = \sqrt{(5-3i)(5+3i)} = \sqrt{34}$.

29. **E** $(2 - i)(2 + i) = 5$, so the point falls on the positive real axis, which falls in none of the quadrants.

30. **D** $w = \left(\frac{1}{2} + i\frac{1}{\sqrt{2}}\right)^5 = \left(\left(\frac{1}{2} + i\frac{1}{\sqrt{2}}\right)^2\right)^2 \left(\frac{1}{2} + i\frac{1}{\sqrt{2}}\right) = \left(\frac{\sqrt{2}}{2}i\right)^2 \left(\frac{1}{2} + i\frac{1}{\sqrt{2}}\right) = \left(-\frac{1}{2}\right)\left(\frac{1}{2} + i\frac{1}{\sqrt{2}}\right) = -\frac{1}{4} - i\frac{1}{2\sqrt{2}}$.
Thus $\Im(w) = -\frac{1}{2\sqrt{2}}$.

TB1 $\boxed{\varnothing}$ Set $z = -\frac{1}{\overline{z}}$, so $z\overline{z} = a^2 + b^2 = -1$. But a, b are both real, so there are no complex z that satisify.

TB2 $\boxed{e^{2\pi^2}}$ We have $i^{4\pi/i} = e^{\frac{i\pi}{2} \cdot \frac{4\pi}{i}} = e^{2\pi^2}$.

TB3 $\boxed{n - 2k}$ Since none of the complex zeros are conjugates of one another, there are at least $2k$ complex zeros, leaving *at most* $n - 2k$ real zeros.