1. **A** - By definition, a function is odd if $f(-x) = -f(x)$, and even if $f(-x) = f(x)$. Upon checking, I. is odd, II. is even, and III. is neither.

2. **E** - Injectivity and surjectivity are characteristics of functions; they do not determine whether or not a relation is a function. On the other hand, a function can have only one output for every input. The way f is defined, $f\left(\frac{1}{2}\right) = 1$, while $f\left(\frac{2}{4}\right) = 2$. Even though the inputs are the same, the outputs differ, and this is not a function.

3. **B** - From Number Theory, $\sigma(n)$ counts the number of divisors of a natural number n.

4. **B** - III. only.

   I. The discontinuity at $x = -2$ is removable. **False**- this is a jump discontinuity.

   II. $\lim\limits_{x\to-2^+} = f(-2)$. **False** - The limit is 4, while $f(-2) = 0$.

   III. $\lim\limits_{x\to 3} f(x)$ exists. **True** - the discontinuity at $x = 3$ is a hole, and thus the limits from the left and right match.

5. **B** - The parabola opens up with the vertex at $(2, -4)$.

6. **B** Taking the absolute value of a function does not affect the location of the zeros (x-intercepts). $f(x) = 12x^3 - 8x^2 - 27x + 18 = (2x-3)(2x+3)(3x-2)$, so the product of the zeros is $\left(\frac{3}{2}\right)\cdot\left(-\frac{3}{2}\right)\cdot\left(\frac{2}{3}\right) = -\frac{3}{2}$

7. **A** - $g(x) = \dfrac{x^3 - 3x^2 - 9x + 27}{x^2 + 2x - 15} = \dfrac{(x-3)^2(x+3)}{(x-3)(x+5)}$. Holes occur when the same factor occurs in the numerator and denominator (as long as the multiplicity is not higher in the denominator!), so the location of the hole is $(3, 0)$.

8. **B** - Note that there is NOT a zero at $x = 3$, because a hole occurs there. Thus, there is only one zero at $x = -3$.

9. **B** - Dividing $x^2+2x-15$ into $x^3-3x^2-9x+27$ and ignoring the remainder, we find the equation of the slant asymptote is $y = x - 5$; ergo the x-intercept is $(5, 0)$.

10. **B** - III. only (again):

    I. $\overline{\omega}$ is a complex zero of f **False** - Complex roots appear in conjugate pairs provided that all coefficients of the function are real. There is no such guarantee here.

    II. f has no more than 3 real zeros **False** - By the same reasons above, f *could* have 4 real zeros.

    III. $\dfrac{\zeta}{\alpha}$ is possible root a real root of f **True** - A result similar to the rational roots theorem provides the necessary results. Recall that the product of roots of a polynomial is $(-1)^n \dfrac{\text{leading coefficient}}{\text{constant term}}$.

11. **C** Breaking down, $\mu(11) = 8$, since "November" has eight letters, and $\ell(8) = 5$ since eight has 5 letters, so $(\ell\circ\mu)(11) = 5$.

12. **C** - We have $\ell(12) = 6$ (since "twelve" has 6 letters), $\ell(6) = 3$, $\ell(3) = 5$, $\ell(5) = 4$, and $\ell(4) = 4$. Thus, $\ell(\ell(\ell(\ell(12)))) = \ell^{(4)}(12) = 4 = \ell^{(k)}(12)$ for all values of $k \geq 4$.

13. **C** - II, III:

    I. β is injective **False** - β is not 1-1 because $\beta(8) = \beta(9) = 0$.

    II. β is surjective **True** - Every integer base eight can we written as, or pulled back to, an integer base ten. Thus we map onto every integer base eight with this function.

    III. β is well-defined **True** - It is impossible to write an integer base ten in a different way to create a different output, so the function is well defined.

14. **B** - The parabola opens up, and all negative functional values occur when $x \in \left(-2 - \frac{\sqrt{44}}{2}, -2 + \frac{\sqrt{44}}{2}\right)$. The *non-positive* integral values in this interval comprise the set $\{-5, -4, -3, -2, -1, 0\}$.

15. **D** - This function has $a = 2$ relative minima at the x-intercepts, $(-5, 0)$ and $(1, 0)$, $b = 1$ relative maximum at $(-2, 3)$, and the product of the zeros is $c = (-5)(1) = -5$. Thus, $a + 2b - c = 9$.

16. **C** - $g(0) = 1$.

17. **B** - Since we must have $\psi(-x) = -\psi(x)$ and $\psi(-x) = \psi(x)$, we have $\psi(x) = -\psi(x)$, and solving yields $\psi(x) = 0$, for any x.

18. **A** Reflecting $f(x) = x^3$ about the line $y = x - 2$ is equivalent to reflecting $g(x) = x^3 + 2$ about the line $y = x$, except we are shifted up 2 units in the second case. We can now find the inverse of g, which is $g^{-1}(x) = \sqrt[3]{x-2}$. Shifting this back down two units, we will find that the desired equation is $\sqrt[3]{x-2} - 2$.

19. **C** - You must think sideways!

    - The function may have more than one y-intercept. **True** - Consider $x = y^2 - 1$.
    - The function must have one or more x-intercepts. **True** - There will always be exactly one x-intercept.
    - The horizontal line test (HLT) will determine if the function is one-to-one. **False** - When x is viewed as a function of y, the roles of the HLT and vertical line test are switched.
    - The range is all real numbers. **False** - When x is viewed as a function of y, only the domain can be $\mathbb{R}$.

20. **D** - We have $\eta(4) = 3$, and if we write $a_n = \eta(n)$, we see that $a_{n+1} = \frac{1}{5}a_n$, so this is a geometric sequence, whose infinite series converges since the common ratio is $\frac{1}{5}$. The starting term in the series is $a_{-1} = \eta(-1) = 9375$, so the indicated sum is $\frac{9375}{1-\frac{1}{5}} = \frac{46875}{4}$.

21. **C** - Since the range of a function is the domain of its inverse, we find that $q^{-1}(x) = \frac{3x+1}{x-2}$, so $x = 2$ is the only value of x not in the range of q.

22. **D** Note that $f(x) + f\left(\frac{1}{x}\right) = x$, and since x is in D, $\frac{1}{x}$ is also in D, we also have $f(\frac{1}{x}) + f\left(\frac{1}{\frac{1}{x}}\right) = \frac{1}{x}$. Since the left-hand sides of both equations are equal, we have $x = \frac{1}{x}$, so the only x's that satisfy are $x = \pm 1$.

23. **E**

    - $X \subseteq Y$ **TRUE** - This must be the case since the composition $(f \circ g)(x)$ is defined - all $x \in X$ must also be in Y, which is the definition of subset.
    - $Y \subseteq X$ **False** - Take $f(x) = g(x) = x$, where $X = [0,1]$ and $Y = [0,2]$ as a counterexample.
    - f and g are inverses **False** - Take the same example as above. To be inverses, the composition must work for all x in the domains of *each* function, but the composition $(f \circ g)(x)$ is not defined for $x \in (1,2]$.
    - both f and g are one-to-one **False** - Take $f(x) = x, g(x) = \lfloor x \rfloor$, where $\lfloor x \rfloor$ is the greatest integer function. Taking $X = Y = \mathbb{R}^+ \cup \{0\}$, the compositions always hold, but g is not 1-1 since it will not pass the horizontal line test.
    - both f and g are continuous **False** - Using the same example as above, we see g is not continuous, yet the compositions hold.

24. **B** A quick sketch of the function shows that the branch of the function containing $(2,3)$ has equation $y = x + 1$, so $a = b = 1$, and $a + b = 2$.

25. **E**

    I. m is rational, not equal to zero **False** - This is true iff the y-intercept is rational.

    II. m is irrational **False** - If a line with irrational slope were to pass through even two lattice points, say (a,b) and (p,q), then the slope of the line would be $\frac{q-b}{p-a}$, which must be rational, creating a contradiction. Thus, a line with irrational slope can pass through at most one lattice point.

    III. $m = 0$ **False** - This is true iff the y-intercept is rational.

26. **B** Again, this problem is best seen with a quick but careful sketch. We find that it is impossible for $x^2 < x^3 < x$.

27. **D** Step I: $f(x)+2$, Step II: $f(x-3)+2$, Step III: $f(-x-3)+2$ and Step IV: $\frac{1}{2}(f(-x-3)+2) = \frac{1}{2}f(-x-3)+1$.

28. **D** Indeterminate form! Here's the stunt: multiply top and bottom by $2x+\sqrt{4x^2-3x+12}$, and cancel the $(x-4)$ factors. Now plug in $x=4$ and obtain $\dfrac{32}{3}$.

29. **B** According to Euler, $e^{ix} = \cos(x) + i\sin(x)$. Thus, $f(z) = e^{2z} = \cos(\frac{2}{i}z) + i\sin(\frac{2}{i}z)$. The period is thus $2\pi \div \dfrac{2}{i} = i\pi$.

30. **A** $x = -\dfrac{b}{2a} = -\dfrac{-12}{2(-2)} = -3$.

TB1 $\lim_{x\to 0^+} \lfloor -x \rfloor = -1$.

TB2 $k=-12$. Set $2^2-4(2)+7 = 3 = \dfrac{k}{3}(2)+11$ and solve for k.

TB3 Note that the range of $-e^x$ is $(-\infty,-1)$ when the domain is $\mathbb{R}^+$ as given, and therefore the range of $\arctan(-e^x)$ must be $(-\frac{\pi}{2}, -\frac{\pi}{4})$.