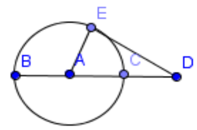
1. $m\angle D = 30^\circ \quad m\angle AED = 90^\circ$

   $D = 2(AE) = 2r \quad BD = 3r$

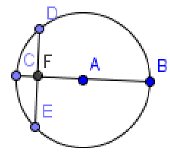
   $6^2 = 3r(r) = 3r^2, \; r = 2\sqrt{3}, \; 3r = 6\sqrt{3}$



2. $DF = EF = 15 \qquad CF = x, \quad BF = 34 - x$

   $15^2 = x(34 - x) \qquad x^2 - 34x + 225 = 0$

   $(x-9)(x-25) = 0 \quad$ smallest is $x = 9$



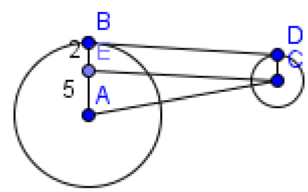
3. By the Pythagorean Theorem, $BC = 4\sqrt{2}$. Area of sector = $\frac{90}{360}\pi(4)^2 = 4\pi$

   Area of □ABC = $\frac{1}{2}(4)(4) = 8$, Area of segment = 4π - 8

4. $CD = BE = 2 \quad AE = 5 \quad CE^2 = 144,$

   $CE^2 + 5^2 = 13^2 \qquad CE = BD = 12$

   $CE^2 + 25 = 169$



5. $(x-4)^2 + (y+8)^2 = r^2 \qquad$ Therefore, $80 - r^2 = -20$

   $x^2 - 8x + 16 + y^2 + 16y + 64 = r^2 \qquad r^2 = 100$

   $x^2 + y^2 - 8x + 16y + (80 - r^2) = 0 \qquad r = 10$

6 $y + 6y + 8y = 360 \qquad m\angle C = \frac{1}{2}(6y - y) = \frac{1}{2}(5y) = \frac{5y}{2} = \frac{5}{2}(24) = 60$

   $15y = 360 \qquad y = 24$

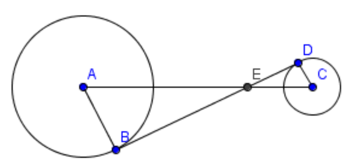
7. Let CD = 8, AB = 12, AC = 30, AE = x, CE = 30 – x;

   $\Delta ABE \sim \Delta CDE, \frac{12}{x} = \frac{8}{30-x}$, 20$x$ = 360, x = 18, 30 – x = 12.

   Applying the Pythagorean Theorem:

   $(BE)^2$ + 144 = 324. BE = $6\sqrt{5}$

   $8^2 + (DE)^2 = 12^2$, $DE = 4\sqrt{5}$, BD = $10\sqrt{5}$.



8. The five arcs are congruent and each measures $\frac{360}{5} = 72$. $m\angle CEG = \frac{1}{2} m\overset{\frown}{CFE} =$

   $\frac{1}{2}(3)(72) = 108 \; m\angle ECF = \frac{1}{2} m\overset{\frown}{EF} = \frac{1}{2}(72) = 36 \qquad m\angle CEG + m\angle ECF = 108 + 36 = 144$

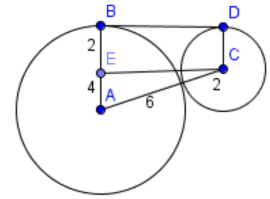
9. Solution: Radii $\overline{AB}$ and $\overline{CD}$ are $\perp$ to $\overleftrightarrow{BD}$.. Draw $\overline{CE}$ $\perp$ $\overline{AB}$.

In 30, 60, 90 rt △ACE, CE = $4\sqrt{3}$, $\therefore$ BD = $4\sqrt{3}$

Area of rectangle CDBE = $(2)\left(4\sqrt{2}\right) = 8\sqrt{3}$

Area of triangle ACE = $\frac{1}{2}(4)\left(4\sqrt{3}\right) = 8\sqrt{3}$

Area of ABDC = $8\sqrt{3}$ + $8\sqrt{3}$ = $16\sqrt{3}$

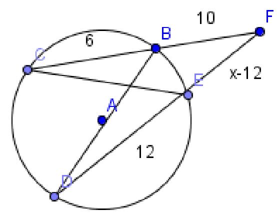


10. $(x-12)x = 16(10)$

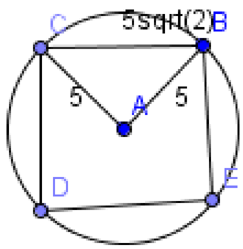
$x^2 - 12x - 160 = 0$

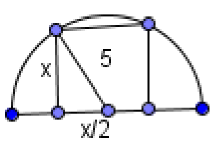
$(x-20)(x+8) = 0$

$x = 20 \quad x = -8$ is unusable



11.





$A_1 = \left(5\sqrt{2}\right)^2 = 50 \quad \frac{x^2}{4} + x^2 = 25, \quad \frac{5x^2}{4} = 25 \quad x^2 = 20 = A_2$

$\frac{A_1}{A_2} = \frac{50}{20} = \frac{5}{2}$

12. Solution: FE = 11 because tangents to a circle from an exterior point are equal.

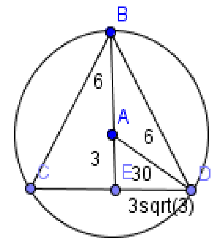
13. Since these are chords intersecting inside the circle,

$(x=1)(7) = 3(x+4), \qquad 4x = 5, \quad x = \frac{5}{4}.$

14. Solution: $m\angle D = \frac{1}{2}(160) = 80°, \quad m\angle FEB = 80°, \quad m\overset{\frown}{BC} = m\overset{\frown}{DE} = 120°,$

$m\overset{\frown}{DC} = 360 - (120 + 120 + 40) = 80 \qquad m\angle F = \frac{1}{2}(80 - 40) = 20$

15. Solution: Area of triangle = $\frac{1}{2}(9)\left(6\sqrt{3}\right)=27\sqrt{3}$



16. $m\angle DBC = 90^\circ - 40^\circ = 50^\circ$, $\triangle DBC$ is isosceles,
$\therefore \angle D = 180^\circ - \left(50^\circ + 50^\circ\right) = 80^\circ$

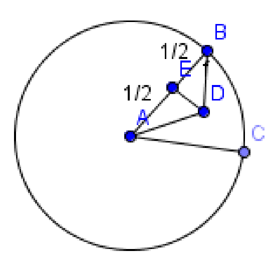
17. Solution: $30 = \frac{1}{2}\left(150 - m\overset{\frown}{BE}\right)$, $60 = 150 - m\overset{\frown}{BE}$, $m\overset{\frown}{BE} = 90$,

$$m\angle CGD = \frac{1}{2}(150+90) = \frac{1}{2}(240) = 120$$

18. Solution: Since the angle between 2 tangents and the central angle are supplementary, $m\angle BAC = 124^\circ$, and the inscribed angle, $m\angle BEC = 62^\circ$, $\triangle BAE \cong \triangle CAE$, and $m\angle BEA = 31^\circ$, $\therefore$ by $\cong$ angles of a isosceles $\triangle$, $m\angle ABE = 31^\circ$

19. Let the radius of the circumscribed circle be r, and the area of the circumscribed circle is πr^2. The triangle is a 45-45-90 triangle and the radius of the inscribed triangle is $\frac{r}{\sqrt{2}}$ and the area of the inscribed circle is $\frac{\pi r^2}{2}$. The ratio is $\frac{\pi r^2}{\frac{\pi r^2}{2}} = \frac{2}{1}$

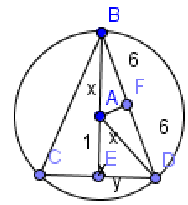
20. Solution:



$\angle BAD = 30^\circ$ by symmetry. Let E be the midpoint of AB.
DA and DB are radii of Circle B and are $\cong$.
$\therefore$ $\triangle ABD$ is isosceles and $\triangle ADE$ is a 30, 60, 90 $\triangle$,
side opp. the 60° $\angle$ = $\frac{1}{2}$ and the hypotenuse = $\frac{\sqrt{3}}{3}$.

21. These are rt. triangles, since they are inscribed in semicircles. The upper triangle has a 2nd leg of 13 by the Pythagorean theorem and the 2nd leg of the lower triangle is 11. The combined areas are $\frac{1}{2}(1)(13)+\frac{1}{2}(7)(11) = \frac{13}{2}+\frac{77}{2} = \frac{90}{2} = 45$.

22. Let the radius of Circle A be r. $(15+r)(15-r) = 4(14)$, $225 - r^2 = 56$,
$r^2 = 169$, $r = 13$, $15 - 13 = 2$



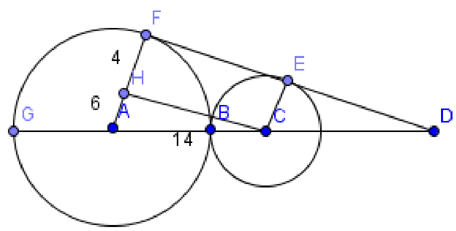
23. $y^2 = x^2 - 1$, $y^2 = 12^2 - (x+1)^2$, $x^2 - 1 = 12^2 - x^2 - 2x - 1$

$2x^2 + 2x - 144 = 0$, $x^2 + x - 72 = 0$, $(x+9)(x-8) = 0$

x = 8 is usable, $y^2 = 64 - 1 = 63$, $y = \sqrt{63} = 3\sqrt{7}$,

perimeter = $12 + 12 + 2\left(3\sqrt{7}\right) = 24 + 6\sqrt{7}$

24. The path makes 4 quarter circles of radius = 2 at the corners and contains four sides of a square that has sides of 5. the length of the path = $2\pi(2) + 4(5) = 4\pi + 20$

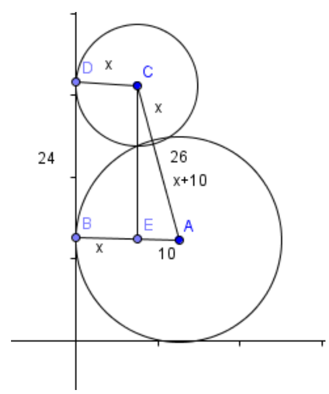


25. $14^2 = 6^2 + (CH)^2$, $160 = (CH)^2$, $CH = 4\sqrt{10}$

by □□s, $\dfrac{DE}{DE + 4\sqrt{10}} = \dfrac{4}{10}$

$4DE + 16\sqrt{10} = 10DE$, $6DE = 16\sqrt{10}$

$DE = \dfrac{16\sqrt{10}}{6} = \dfrac{8\sqrt{10}}{3}$



26. Solution: $10 + 2x = 26 \Rightarrow 2x = 16 \Rightarrow x = 8$

The coordinates of C are $(8, 24 + 18)$ or $(8, 42)$

The coordinates of A are $(18, 18)$

27. The center of the circle is $(2,-3)$. The slope of the radius to $(8,-8)$ is m = $\dfrac{-8+3}{8-2} = \dfrac{-5}{6}$.

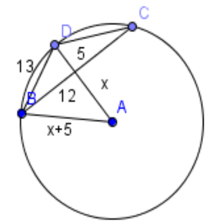
The slope of the tangent at $(8,-8) = \dfrac{6}{5}$. The equation of the tangent is

$(y+8) = \dfrac{6}{5}(x-8) \Rightarrow y = \dfrac{6}{5}x - \dfrac{88}{5}$

28. The grazing area consists of $\dfrac{3}{4}$ of a circle with radius 100 and

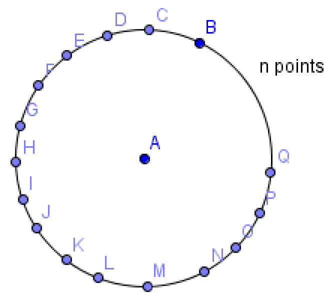
$2\left(\dfrac{1}{4} \text{ of circle of radius } 20\right) \Rightarrow A = \dfrac{3}{4}(\pi)(10000) + \dfrac{1}{2}(\pi)(400) = 7700\pi$

29. Solution: $x^2 + 12^2 = (x+5)^2 \Rightarrow x^2 + 144 = x^2 + 10x + 25 \Rightarrow$
$10x = 119 \quad x = 11.9 \Rightarrow x + 5 = 16.9$

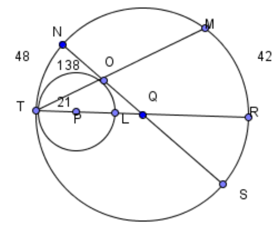


30. From every point of the n points an arc can be named to n-1 points, but a major and minor arc can be named in each case.

So, there are $2\dfrac{n(n-1)}{2} = n(n-1)$



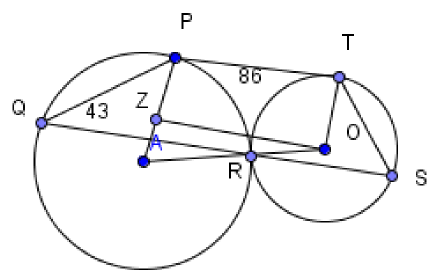
Tiebreaker 1: $m\angle MTR = 21, \quad m\overset{\frown}{OL} = 42, \quad m\angle NQT =$
$\dfrac{1}{2}(138-42) = \dfrac{1}{2}(96) = 48.\ m\overset{\frown}{TN} = 48,\ m\overset{\frown}{MN} = 180 - 42 - 48 = 90$



Tiebreaker 2.

Using the common tangent procedure,

by inscribed angles, $m\overset{\frown}{PR} = 86° \Rightarrow$
$m\angle PAR = 86 \Rightarrow \quad m\angle ZOA = 4° \quad \Rightarrow$
$m\angle ROT = 90° + 4° = 94°$
$m\angle S = \dfrac{1}{2}(94) = 47°$



Tiebreaker 3. In the smaller circle, 70 = $\dfrac{1}{2}\left(m\overset{\frown}{BD} + 20\right) \Rightarrow \dfrac{1}{2}m\overset{\frown}{BD} = 60 \Rightarrow m\overset{\frown}{BD} = 120.$

In the larger circle, 70 = $\dfrac{1}{2}\left(160 - m\overset{\frown}{BD}\right) \Rightarrow 140 = 160 - m\overset{\frown}{BD} \Rightarrow m\overset{\frown}{BD} = 20$

$\therefore 120 - 20 = 100$