Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. What is the shape of the conic with equation $3x^2 - 500x - 17y^2 + 600y = 2009$?

(A) Circle (B) Parabola (C) Hyperbola (D) Catenary (E) NOTA

2. Given that $x$, $y$, and $z$ satisfy the following system:

$$\begin{aligned} 3x - 4y + z &= -7 \\ 2x + 2y - 3z &= 12 \\ -x + 3y + 2z &= 1 \end{aligned}$$

Find the value of $\left(\frac{y}{x}\right)^2 - z^2$.

(A) $-2$ (B) 0 (C) 4 (D) 8 (E) NOTA

3. Let $y = L(x)$ be the equation of the line tangent to the graph of $y = x^3 + 2x + e^{x-1}$ at the point (1, 4). Find the value of $L(10)$.

(A) 22 (B) 34 (C) 46 (D) 58 (E) NOTA

4. Let $i = \sqrt{-1}$. Evaluate: $i^6 + i^{34} + i^{58} + i^{65} + i^{76}$

(A) $-2 + i$ (B) $4 + i$ (C) $3 - 2i$ (D) $-i$ (E) NOTA

5. Find the sum of all solutions to $\sin^2\theta = \frac{5}{13}$ on the interval $0 \le \theta \le 4\pi$.

(A) $6\pi$ (B) $8\pi$ (C) $10\pi$ (D) $12\pi$ (E) NOTA

6. Find the sum of the coordinates of the local minimum of the graph of $y = 3x^3 - 9x + 1$.

(A) 4 (B) 0 (C) $-1$ (D) $-4$ (E) NOTA

7. The graph of $y = f(x) = ax^2 + bx + c$ passes through the points (1, 4), (3, 14), and (5, 40). Find the value of $f(7)$.

(A) 56 (B) 69 (C) 82 (D) 95 (E) NOTA

8. Find the determinant of $\begin{pmatrix} 0 & 1 & 4 \\ 7 & 2 & 6 \\ 4 & -8 & 5 \end{pmatrix}$.

(A) $-267$ (B) $-15$ (C) 7 (D) 104 (E) NOTA

9. Find the value $c$ that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = x^3 + 3x^2$ on the interval $-5 \le x \le 1$.

(A) 0 (B) $-2$ (C) $-1$ (D) $-3$ (E) NOTA

10. What is the remainder when $10^{2009}$ is divided by 7?

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

11. A triangle with side lengths 7 meters, 15 meters, and 20 meters has a perimeter of $P$ meters and area of $A$ square meters. Find the value of $A + P$.

(A) 58 (B) 65 (C) 87 (D) 150 (E) NOTA

12. Evaluate: $\int_0^2 \left(2x^2 - 3x + 2\right) dx$

(A) 4 (B) $\frac{11}{3}$ (C) 5 (D) $\frac{10}{3}$ (E) NOTA

13. A sequence $a$ is defined recursively as $a_0 = 0, a_1 = 1$, and for $n > 1, a_{n+1} = 3a_n - 2a_{n-1}$. Find the sum of the squares of the four smallest positive values of $K$ such that $a_k$ is prime.

(A) 39 (B) 87 (C) 207 (D) 367 (E) NOTA

14. It can be shown that $p = 15463339$ is prime. What is the smallest prime number greater than $p$?

(A) 15463341 (B) 15463343 (C) 15463349 (D) 15463359 (E) NOTA

15. A balloon in the shape of a sphere is being inflated such that it maintains its spherical shape. The volume of the balloon is increasing at a rate of $3\pi$ cubic meters per second. How fast, in meters per second, is the radius of the balloon changing the moment the volume is $\frac{9\pi}{2}$ cubic meters?

(A) $\frac{1}{3}$ (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) 3 (E) NOTA

16. What is the sum of the reciprocals of the positive integral divisors of 360?

(A) 2 (B) $\frac{15}{2}$ (C) $\frac{13}{4}$ (D) $\frac{25}{6}$ (E) NOTA

17. A particle is moving along the $x$-axis such that its position $p$ at time $t$ is given by $p(t) = t + 32t^{-2}$ for $t \geq 0$. What is the acceleration of the particle when its velocity is 0?

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{5}{6}$ (D) $\frac{3}{4}$ (E) NOTA

18. How many ways can seventeen lines (no two parallel, no three concurrent) divide a plane into distinct regions?

(A) 154 (B) 145 (C) 136 (D) 127 (E) NOTA

19. The base of a solid $S$ is the region bounded by the graphs of the x-axis and $y = (2 - x)\sqrt{x}$. The cross sections of $S$ perpendicular to the $x$-axis are in the shape of equilateral triangles. Find the volume of $S$.

(A) $\frac{16\sqrt{2}}{15}$ (B) $\frac{5\sqrt{2}}{8}$ (C) $\frac{2\sqrt{3}}{5}$ (D) $\frac{\sqrt{3}}{3}$ (E) NOTA

20. The value of $\sum_{n=1}^{\infty} \frac{1}{(3n-2)(3n+1)}$ is equal to $\frac{m}{n}$, where $m$ and $n$ are relatively prime natural numbers. Find the value of $m^2 + n^2$.

(A) 5 (B) 13 (C) 17 (D) 9 (E) NOTA

21. For this problem, use the following table. Assume that $f$ and $g$ are differentiable for all $x$ and $f' > 0$ for all $x$.

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 1 | 0 | 1 | 5 | 8 |
| 2 | 5 | 4 | 3 | –6 |
| 3 | 8 | 2 | –3 | 4 |
| 4 | 11 | 7 | 1 | –1 |

Let $h(x) = f^{-1}(g(x))$. Evaluate: $h'(1)$.

(A) 2 (B) 4 (C) 8 (D) 16 (E) NOTA

22. Let $a, b, c$ and $d$ in that order, represent a permutation of the four smallest positive integers. Let $f(a,b,c,d) = (a-b)^2 + (b-c)^2 + (c-d)^2 + (d-a)^2$. Let $M$ equal the largest possible value of $f$ and $m$ equal the smallest possible value of $f$. Find the value of $Mm$.

(A) 60 (B) 120 (C) 180 (D) 240 (E) NOTA

23. Let $R$ be the region bounded by the graphs of $y = 1 - x^2$ and $y = 0$. Find the volume of the solid formed when $R$ is revolved about the line $y = x - 2$.

(A) $\frac{32\pi\sqrt{2}}{15}$ (B) $\frac{7\pi\sqrt{3}}{3}$ (C) $\frac{28\pi\sqrt{3}}{9}$ (D) $\frac{16\pi\sqrt{2}}{5}$ (E) NOTA

24. Let $S = \{3, 3^2, 3^3, ..., 3^{50}\}$. How many ways can two distinct elements $a$ and $b$ be randomly chosen from $S$ such that $\log_b a$ is *not* an integer?

(A) 2359 (B) 2293 (C) 2053 (D) 1936 (E) NOTA

25. Let $P$ be a polynomial function such that $P(x) < 0, P'(x) > 0$, and $P''(x) > 0$ for all $x \in [-1, 11]$. Define the following quantities:

- $I = \int_0^{10} P(x)\,dx$
- $L$ = Approximation of $I$ using a left-hand Riemann sum of 2009 equal partitions.
- $R$ = Approximation of $I$ using a right-hand Riemann sum of 2009 equal partitions.
- $T$ = Approximation of $I$ using the Trapezoid Rule of 2009 equal partitions.

Rank the quantities above in order from largest to smallest.

(A) $R > I > T > L$ (B) $L > I > T > R$

(C) $L > T > I > R$ (D) $R > T > I > L$ (E) NOTA

26. Richard keeps tossing a fair, two-sided coin until he obtains two heads in a row immediately followed by a tail (i.e., head-head-tail). What is the expected number of times he tosses the coin?

(A) 8 (B) 7 (C) 10 (D) 5 (E) NOTA

27. The value of $\sum_{k=0}^{\infty} \frac{k^2}{4^k}$ can be written as $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. Find the value of $m + n$.

(A) 28 (B) 47 (C) 66 (D) 85 (E) NOTA

28. In triangle ABC, |AB| = 5, |AC| = 4, and |BC| = 3. Let D be a point on side AB such that |BD| = 2. The value of |CD| can be expressed as $\frac{a + b\sqrt{p}}{c}$, where $a$, $b$, $c$, and $p$ are positive integers such that $p$ is prime and the greatest common factor of all the four numbers is 1. Find the value of $|a| + |b| + |c| + |p|$.

(A) 37 (B) 36 (C) 35 (D) 34 (E) NOTA

29. Let $f_0(x), f_1(x), f_2(x), f_3(x), \ldots$ be a sequence of polynomials such that:

- $f_0(x) = 1$ for all $x$.
- $f_n(0) = 0$ for integers $n \geq 1$ .
- $\frac{df_n}{dx} = nf_{n-1}(x+1)$.

Find the number of positive integral factors of $f_{150(13)}$.

(A) 100 (B) 200 (C) 300 (D) 400 (E) NOTA

30. Two teams with five members per team are competing in a running contest. The contest is set so that the *n*th place finisher contributes *n* points to their team's score. The team with the lower score wins the contest. Assuming no ties, how many winning scores are possible?

(A) 16 (B) 15 (C) 13 (D) 14 (E) NOTA