

**ANSWERS:**

1. 1	6. $\frac{1}{32}$	11. 6	16. $\frac{1}{k}$	21. $\frac{11}{6}$
2. $\frac{1}{2}$	7. $-\frac{\sqrt{2}}{4}$	12. 1	17. $-\frac{1}{3}$ *	22. $-\frac{3}{8}$
3. -1	8. 48	13. 4	18. 26	23. 16
4. -5	9. -1	14. 5	19. 5	24. 4
5. -1	10. 4	15. $\frac{1}{4}$	20. $\frac{7}{3}$	25. $-\frac{1}{2}$

\* one value only.

**SOLUTIONS:**

1.  $f'' = 12x - 3 > 0$  for  $x > 0.25$ , and the least integer is 1.

2.  $\int_0^{\pi/4} \sec^2 x \tan x dx = \int u du$  for  $u = \tan x$ .  $\frac{1}{2} \tan^2 x \Big|_0^{\pi/4} = \frac{1}{2}(1 - 0) = 1/2$ .

3.  $f = \sqrt{(x-1)^2} = |x-1|$ , and the slope to the left of the cusp is -1.

4. The derivative of  $y = 2x^2 + 3x$  is  $4x+3$ . At  $x = -2$ , the derivative is -5.

5.  $2xe^{2x} + x^2(2e^{2x}) < 0$ ,  $2xe^{2x}(1+x) < 0$ . Between  $x=0$  and  $x=-1$  this is true, and so  $a = -1$ .

6.  $\frac{dy}{dt} = 8x \frac{dx}{dt}$ ,  $\frac{dy}{dt} = 8x \left( 4 \frac{dy}{dt} \right)$ . This is true for  $x = 1/32$ .

7.  $f' = \frac{1}{2}(9-x^2)^{-1/2}(-2x)$  and at  $x = 2\sqrt{2}$ ,  $f' = -2\sqrt{2}$ . so  $g'(1) = 1/f'(2\sqrt{2}) = -\frac{\sqrt{2}}{4}$ .

8.  $f' = 4(g(x))g'(x)$  by the chain rule, and  $4(4)(3) = 48$ .

9.  $f'' = 6x+3 < 0$  and  $f' < 0$  for the interval  $(-2, -1/2)$  and the integer in this interval is -1.

10.  $\frac{1}{6} \left( -\frac{3}{4}x^2 + kx \right) \Big|_0^6 = -\frac{1}{2}$  when  $(-27+6k) = -3$  and  $k = 4$ .

11. The values must be equal at  $x=1$  so  $a+3+b = 2a-b$  and  $-a+2b = -3$ . The derivatives must be equal  $2a+3 = 2a-b$  so  $b = -3$ , and thus  $a = -3$ . So the abs value of  $a+b$  is 6.

12.  $e^x = e$  when  $x=1$ , so the area is  $\int_0^1 (e - e^x) dx = ex - e^x \Big|_0^1 = (e - e) - (0 - 1) = 1$ .

13.  $y dy = dx$ ,  $\frac{y^2}{2} = x + c$ ,  $2 = -1 + c$ ,  $c = 3$ . For the initial condition, we have the function is the

"positive" part of the graph  $y = \sqrt{2x+6}$ . When  $x=5$   $y=4$ .

14. The values of  $f'$  are 0 or undefined at  $x = 0, \pi, \pi/2, 3\pi/2, 2\pi$ . Five values.

15.  $8x+3 = 9-16x$  at  $x = 1/4$ .

16.  $f'$  is zero and  $f''$  must be positive.  $f'=0$  at  $x=1/k$  or  $x=-2k$ .  $f'' = kx^2 + 2k^2x - x - 2k$ . When  $x = 1/k$ ,  $f'' = 2 + 2k^2 - 1$  which is positive. At  $x = -2k$ ,  $f'' = -2k^2 - 1$  which is negative so the  $x$ -coordinate of the rel. min is  $1/k$ .

17.  $f(-1) = -2$  and  $f(1) = 0$ , which gives slope 1.  $f' = 3x^2 - 2x = 1$  when  $x = -1/3$  or  $x = 1$ , but the MVT says that the value of  $x=1$  is not part of the conclusion, so  $x = -1/3$ .

18.  $3(6) + \int_0^2 4dx = 18+8 = 26$ .

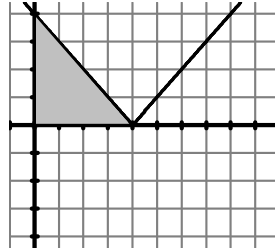
19. For  $f(x) = 3kx+1$ ,  $f(1)+f(2)+f(3)+f(4) = 3k+1 + (6k+1) + (9k+1) + (12k+1) = 30k+4 = 154$  gives  $k=5$ .

20.  $u = x^2 + 1, du = 2x dx$ ,  $\int_1^4 u^{1/2} (.5 du) = \frac{1}{2} \cdot \frac{2}{3} u^{3/2} \Big|_1^4 = \frac{1}{3}(8-1) = 7/3$ .

21.  $f(2) = 1$  and tangent line has slope  $6x-6 = 6$ .  $y-1 = 6(x-2)$  has  $x$ -intercept  $11/6$ .

22.  $m = 2(4)/3 = 8/3$ . Normal slope =  $-3/8$ .

23. The area described by the integral is two triangles like the shaded one shown. So the integral gives area  $2(1/2)(4)(4) = 16$ .



24.  $f''=1$  means  $f' = x+c$  and  $f = \frac{1}{2}x^2 + cx + k$

and when  $x=1$  we get  $0.5+c+k=4$ .

When  $x= -1$ , we get  $0.5 - c+k= 5$ . Add to get

$1+2k = 9$  and so  $k=4$ . So  $f(0) = k = 4$ .

25. Every fourth derivative is  $\cos x$  so the 40<sup>th</sup> derivative is  $\cos x$ .  $f^{(41)}(x) = -\sin x = -\sin \frac{\pi}{6} = -\frac{1}{2}$ .