

For all questions, note the following:

(1) For choice E, NOTA stands for "None Of The Above" answers is correct.

(2) $i = \sqrt{-1}$

1. Evaluate $7 + 3 \cdot 2 - 10 \div 5$

- A. -16 B. $\frac{3}{5}$ C. 2 D. 11 E. NOTA

2. How many positive prime factors does 2010 have?

- A. 1 B. 2 C. 3 D. 4 E. NOTA

3. A table of values for an operation "@" is shown below.

| @ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 |
| 2 | 2 | 4 | 1 | 3 | 5 |
| 3 | 3 | 1 | 4 | 2 | 5 |
| 4 | 4 | 3 | 2 | 1 | 5 |
| 5 | 5 | 5 | 5 | 5 | 5 |

Which number is the identity for the operation @?

- A. 1 B. 2 C. 4 D. 5 E. NOTA

4. Let k be the solution to the equation $9^{2x+5} = 27^{7x-11}$. If $|k| = \frac{P}{Q}$, where $\frac{P}{Q}$ is a fraction in lowest terms, then what is the value of $P+Q$?

- A. 40 B. 48 C. 60 D. 68 E. NOTA

5. How many integer values of x satisfy the inequality $\frac{x-3}{11-x} \geq 1$?

- A. 3 B. 4 C. 5 D. 6 E. NOTA

6. Evaluate $\begin{vmatrix} -3 & 5 & 1 \\ 2 & 1 & 4 \\ -2 & -1 & 3 \end{vmatrix}$

- A. -91 B. -11 C. 11 D. 91 E. NOTA

7. A jar contains 2010 marbles. One-third of the marbles are red, one-half of them are blue, 100 of them are yellow, and the rest of them are green. If a marble is selected at random, the probability of selecting a green marble can be written as $\frac{K}{402}$ for a positive integer K . What is the sum of the digits of K ?

- A. 5 B. 7 C. 9 D. 11 E. NOTA

8. The graph of $f(x) = \lfloor 2x + 5 \rfloor$ (where $\lfloor x \rfloor$ represents the greatest integer function) is made up of an infinite number of line segments that each have what length?

- A. $\frac{1}{5}$ B. $\frac{1}{2}$ C. 2 D. 5 E. NOTA

9. What is the units digit of $2^{2010} + 3^{2010} + 5^{2010}$?

- A. 2 B. 4 C. 6 D. 8 E. NOTA

10. The function $f(x) = 2x^4 - x^3 - 23x^2 + 46x - 24$ has four distinct real roots P , Q , R , and S , where $P < Q < R < S$. What is the value of $P^Q + RS$?

- A. -1 B. 4 C. 7 D. 8 E. NOTA

11. A student class consists of ten students, made up of six girls and four boys. The class is holding elections for the offices of president, vice-president, and secretary. In how many different ways can three of these students occupy the offices if there must be at least one girl and one boy among the three officers?

- A. 576 B. 600 C. 696 D. 720 E. NOTA

12. If $\frac{2}{1+i+\sqrt{5}} = \frac{P+Q\sqrt{5}+(R-4\sqrt{5})i}{-29}$ for integers P , Q , and R , then what is the value of $P+Q+R$?
- A. -18 B. 2 C. 10 D. 30 E. NOTA
13. A jar containing ten pounds of nuts is made up of six pounds of walnuts and four pounds of cashews. How many pounds of only walnuts must be added to this jar so that 75% of the weight inside the jar is made up of walnuts?
- A. 6 B. 10 C. 12 D. 16 E. NOTA
14. If $f\left(\frac{x+2}{x}\right) = 5+2x-x^2$, then what is the value of $f(3)$?
- A. 2 B. 6 C. 8 D. 20 E. NOTA
15. At Camembert High School, 100 students were surveyed on what type of cheeses they like, of American, cheddar, and Swiss. The results are listed below:
- | | |
|--------------------------------------|------------------------------|
| 64 like American | 70 like cheddar |
| 56 like Swiss | 45 like American and cheddar |
| 40 like cheddar and Swiss | 47 like American and Swiss |
| 36 like American, cheddar, and Swiss | |
- How many of the 100 students do not like any of the three cheeses?
- A. 4 B. 5 C. 6 D. 8 E. NOTA
16. What is the sum of the digits of the 2010th term of the arithmetic sequence whose first five terms are 7, 11, 15, 19, and 23?
- A. 15 B. 16 C. 19 D. 20 E. NOTA
17. For real numbers x and y , if $x+y=10$ and $xy=20$, then what is the value of x^3+y^3 ?
- A. 400 B. 700 C. 1000 D. 1300 E. NOTA

18. Evaluate $\prod_{n=1}^{2010} i^n$

- A. -1 B. 1 C. $-i$ D. i E. NOTA

19. What is the distance between the centers of the graphs of $x^2 + y^2 - 6x + 8y - 2010 = 0$ and $2x^2 - 3y^2 + 20x + 36y - 2010 = 0$?

- A. $2\sqrt{2}$ B. $2\sqrt{17}$ C. $2\sqrt{26}$ D. $2\sqrt{41}$ E. NOTA

20. A ball is dropped from a height of 60 feet. Each time the ball hits the ground, it rebounds to $\frac{2}{3}$ of its previous height. At the exact instant that the ball hits the ground the second time, a flat piece of wood is placed 10 feet above the ground and acts as a "ceiling" that the ball hits on the way back up. Once the ball hits the piece of wood, it falls 10 feet and hits the ground, then continues to rebound to $\frac{2}{3}$ its previous height.

What is the total vertical distance, in feet, traveled by the ball?

- A. 130 B. 160 C. 170 D. 200 E. NOTA

21. Given the recursive function f defined by

$$f(1) = 3 \quad \text{and} \quad f(n) = 2f(n-1) + 7 \quad \text{for integer values of } n$$

What is the sum of the digits of $f(8)$?

- A. 10 B. 12 C. 13 D. 15 E. NOTA

22. A square with area 1 square unit has a circle inscribed inside. A smaller square is inscribed in this circle. What is the perimeter of the smaller square?

- A. $\frac{1}{4}$ B. $\sqrt{2}$ C. $2\sqrt{2}$ D. $4\sqrt{2}$ E. NOTA

23. Evaluate $12 - \sqrt{12 - \sqrt{12 - \sqrt{12 - \dots}}}$

- A. 2 B. 4 C. 6 D. 8 E. NOTA

24. The graph of the function $f(x) = \frac{3x^2 + x + 4}{x^2 - 6x + 8}$ has asymptotes $x = P$, $x = Q$, and $y = R$, where $P < Q$. What is the value of $P^R + Q$?

- A. 12 B. 13 C. 19 D. 66 E. NOTA

25. How many integer values for x satisfy the equation $(x^2 - 7x + 11)^{(x^2 + 17x + 72)} = 1$?

- A. 2 B. 3 C. 4 D. 5 E. NOTA

26. If $f(x) = \begin{cases} 4 - 3x, & x < -2 \\ x^3, & -2 \leq x < 1 \\ x^2 + 7, & x > 1 \end{cases}$, then what is the value of $f(f(-2))$?

- A. -20 B. 28 C. 71 D. 107 E. NOTA

27. If $\log 2 = a$ and $\log 7 = b$, then what is $\log \frac{343}{16}$ in terms of a and b ?

- A. $\frac{3b}{4a}$ B. $\frac{b^3}{a^4}$ C. $3b - 4a$ D. $b^3 - a^4$ E. NOTA

28. The population P of raccoons in a designated area at time $t \geq 0$ (in years) is given by the equation

$$P(t) = \frac{100}{1 + 3e^{-0.1t}}$$

where $t = 0$ represents the start of the year 2000. Which of the following are true?

- I. The population at the beginning of 2000 is 25 raccoons.
 II. The population will never reach 150 raccoons.
 III. The population never decreases.

- A. I and II only B. I and III only C. II and III only
 D. I, II and III E. NOTA

29. What is the area of the region bounded between the graphs of $y = |x|$ and $y = 4 - |x|$, but also above the line $y = 1$?

- A. 2 B. 3 C. 6 D. 7 E. NOTA

30. In the binomial expansion of $(4x + y)^{\frac{1}{2}}$, what is the coefficient of the fourth term?

- A. $\frac{1}{4}$ B. $\frac{1}{64}$ C. $\frac{1}{128}$ D. $\frac{1}{512}$ E. NOTA