

Choose the best response for each of the following questions. For all questions, answer E. "NOTA" means "none of these answers is correct." NO calculators are permitted this test. (Note: \mathbb{R} denotes the "set of Real Numbers.")

1. Express the following expression in simplest form: $\left(\sqrt{\sqrt[3]{256} - \sqrt[3]{108}}\right)^{-3}$.

- A. $-\frac{1}{2}$ B. $\frac{1}{5}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. NOTA

2. Find the value of r for which $\log(ab) + \log\left(\frac{1}{ab}\right) = \log_2 r$, and $a > 0, b > 0$.

- A. 1 B. 2 C. 3 D. 4 E. NOTA

3. If x and y are solutions of the following system, find the quantity of $y - x$

$$\begin{cases} \log_8 1 + \log_3(x + 2) = \log_3(3 - 2y) \\ 2^{x+y} - 8^{3-y} = 0 \end{cases}$$

- A. -7 B. -3 C. 5 D. 11 E. NOTA

4. Simplify the expression to evaluate: $\frac{1}{\log_4 18} + \frac{1}{2\log_6 3 + \log_6 2} + \frac{5}{\log_3 18} =$

- A. 1 B. 2 C. 3 D. 4 E. NOTA

5. If $S = \log_2(\sqrt{7} + \sqrt{5})$, express $\log_2(\sqrt{7} - \sqrt{5})$ in terms of S.

- A. $\frac{2}{S}$ B. $\frac{S-1}{S}$ C. $1-S$ D. $2-S$ E. NOTA

6. Find all the ordered pairs of (x, y), $x, y \in \mathbb{R}$ such that $3^{x^2-2xy} = 1$ and $2\log_3 x = \log_3(y+3)$.

- A. $\left(0, -\frac{3}{4}\right)$ B. $\left(0, \frac{3}{2}\right)$ C. (2, 1) D. $\left(2, -\frac{3}{4}\right)$ E. NOTA

7. If $\log_b 125 = c$, then $\log_b 25$ is what percent of the value of c?

- A. 20 B. $33\frac{1}{3}$ C. 50 D. $66\frac{2}{3}$ E. NOTA

8. Given $x > 1$ and $\log_x x^{x^2} + \log_x x^{-5x} = \log_x \left(\frac{1}{x^6} \right)$, find all values of x that satisfy the equation.
 A. $\{-6, 1\}$ B. $\{2, 3\}$ C. $\{4\}$ D. $\{1, 6\}$ E. NOTA
9. Find the value of x , $x \in \left(0, \frac{\pi}{2} \right)$ and $\ln(\sin x) - \ln(\cos x) = 1$.
 A. $\{ \}$ B. $\arctan \pi$ C. $\arcsin e$ D. $\arctan e$ E. NOTA
10. Solve for y given that $y > 2$ and $\sin x > 0$ for the equation $\ln(y-2) = \ln(\sin x) - 3x$.
 A. $-e^{-3x} \sin x - 2$ B. $e^{-3x} \sin x - 2$ C. $e^{3x} \sin x + 2$ D. $e^{-3x} \sin x + 2$ E. NOTA
11. How many real solutions are there for x in: $\log_3 x = 2 \cos x$?
 A. 0 B. 1 C. 2 D. 3 E. NOTA
12. For $0 < x < \pi$, $3^{\tan x} = 27^{\sin x}$. Find $\cos x$.
 A. 0 B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. 3 E. NOTA
13. Find the sum of all the values of x that satisfy: $4^x - 3^2 \cdot 2^x + 2^3 = 0$.
 A. 0 B. 2 C. 3 D. 8 E. NOTA
14. Find $f^{-1}(x)$ when $f(x) = \log(x+2)^3$ where both functions are defined.
 A. $\frac{1}{\log(x+2)^3}$ B. $\sqrt[3]{10^x - 2}$ C. $\sqrt[3]{\log(x+2)}$ D. $\sqrt[3]{10^x - 2}$ E. NOTA
15. Given the expression $\left(\frac{54}{111} \right)_x = \left(\frac{16}{25} \right)_x$ and positive integer values of base x , find x .
 A. 7 B. 8 C. 9 D. 11 E. NOTA

16. Uncle Sam is told that $a = x - 2$ and $b = x - 4$ and $a > 2$, $b > 2$. Find x such that

$$\frac{\log_a(x-3) \log_b(x+10)}{\log_b(x-3)} = 2.$$

- A. 2 B. 3 C. 6 D. $\{2, 3\}$ E. NOTA
17. Find the real values of x so that the values of all terms are real and satisfy the equation $\sqrt{2x} = \sqrt{x+7} - 1$.
- A. 2 B. 12 C. $\{12, 18\}$ D. $\{2, 18\}$ E. NOTA

18. Given that $\frac{1}{\log_7 2} + \frac{1}{\log_9 4} = x$, find the value of the expression 4^x .

- A. 21 B. 24 C. 441 D. 576 E. NOTA

19. Find the coefficient of d^{-10} in the expansion of $\left(d^4 - \frac{2}{d^3}\right)^{15}$.

- A. -1365 B. 3003 C. 5005 D. 6006 E. NOTA

20. Which of the following coordinate pairs (x, y) satisfies the system:
$$\begin{cases} \log_x 9 + \log_8 y = \frac{7}{3} \\ \log_9 x + \log_y 8 = \frac{7}{2} \end{cases}$$

- A. $\left(\frac{1}{2}, 3\right)$ B. $\left(2, \frac{1}{3}\right)$ C. $(8, 9)$ D. $(729, 64)$ E. NOTA

21. Find the domain of the equation $f(x) = \sqrt{\frac{4}{x-2}}$.

- A. $x \in \mathbb{R}$ B. $x \geq 2$ C. $x > 3$ D. $x > 4$ E. NOTA

22. The expression $\ln(\log_7 35 - \log_7 5)$ simplifies to $__? __$.

- A. -1 B. 0 C. 1 D. $-\ln 7$ E. NOTA

23. Careful multiplication of the infinite number of terms will simplify the product of

$$\left(2^{\frac{1}{3}}\right)\left(4^{\frac{1}{9}}\right)\left(8^{\frac{1}{27}}\right)\left(16^{\frac{1}{81}}\right)\dots \text{ to be } __? __.$$

- A. $2^{\frac{1}{4}}$ B. $2^{\frac{1}{2}}$ C. $2^{\frac{3}{4}}$ D. $2^{\frac{5}{4}}$ E. NOTA

24. Honest Abe thought he saw this problem on the first page, but careful analysis led him find the correct solution for x and y . Find the sum of x and y that Abe found for this system.

$$\begin{cases} 2^{x-y} - 8^{3-y} = 0 \\ \log_3 1 + \log_5 (x+2) = \log_5 (2y-3) \end{cases}$$

- A. 5.5 B. 4.5 C. 1.5 D. -1.5 E. NOTA
25. What is the sum of the coefficients containing c^3 in the simplified expansion of $(2a - 3b + c)^6$?
- A. -2010 B. -20 C. 0 D. 20 E. NOTA
26. The solution x of the equation $\log_4 (3x + 7) - \log_4 (x - 5) = 2$ would lie within which of the given ranges?
- A. $0 \leq x \leq 3$ B. $3 \leq x \leq 6$ C. $6 \leq x \leq 9$ D. $9 \leq x \leq 12$ E. NOTA
27. Simplify: $\sum_{k=1}^{\infty} \frac{3(4^{k-1})}{5^k} =$
- A. ∞ B. 12 C. 9 D. 3 E. NOTA
28. Given that $a^{2b} = 5$, find $2a^{6b} - 4$.
- A. 240 B. 246 C. 996 D. 2010 E. NOTA
29. Find the real value of x that makes the equation $\frac{e^x + e^{-x}}{e^x - e^{-x}} = 2$ true.
- A. $0.5 \ln 3$ B. 1 C. $2 \ln 3$ D. $\ln 3$ E. NOTA
30. The best part of any test is getting the last question correct. Simplify $\frac{2^{-1}(16x^5)^{1/4}}{2x^{1/4}}$ for $x > 0$.
- A. $0.25x$ B. $0.5x$ C. 0.5 D. $x^{3/2}$ E. NOTA