

1. **D** The first three nonzero terms in question are $x - x^3/6 + x^5/120$. Plugging in, we get

$$\int_0^1 (1 - x^2/6 + x^4/120)dx = 1 - \frac{1}{18} + \frac{1}{600} = \frac{1703}{1800}.$$

2. **B** Using the Ratio Test, we get $\lim_{x \rightarrow \infty} \frac{(2x+2)!}{(x+1)^{x+1}(x+10)!} \times \frac{x!x^x}{(2x)!} = \lim_{x \rightarrow \infty} \frac{(2x+2)(2x+1)x^x}{(x+1)^2(x+1)^x}$
 $= \lim_{x \rightarrow \infty} \frac{2(2x+1)}{x+1} \times \left(\frac{x}{x+1}\right)^x = \lim_{x \rightarrow \infty} \frac{4x+2}{x+1} \left(1 + \frac{1}{x}\right)^{-x} = \frac{4}{e} > 1$ and so our series diverges.

3. **C** Rewrite this as $r^2 = \frac{25}{4 \cos^2(\theta) + 9 \sin^2(\theta)} \rightarrow 4(r \cos(\theta))^2 + 9(r \sin(\theta))^2 = 25 \rightarrow 4x^2 + 9y^2 = 25$.

Write this ellipse in standard form: $\frac{x^2}{\frac{25}{4}} + \frac{y^2}{\frac{25}{9}} = 1$; the area is thus $\pi(\frac{5}{2})(\frac{5}{3}) = \frac{25\pi}{6}$.

4. **B** $A = \pi(a)(7-a)$; the average value is $\frac{\pi}{7} \int_0^7 (7a - a^2)da = \frac{\pi}{7}(\frac{343}{2} - \frac{343}{3}) = \frac{49\pi}{6}$.

5. **E** It says nothing about a *local* minimum. As $t \rightarrow -\infty$, so too does this dot product.

6. **C** $x = r \cos(\theta) = 2 \cos(\theta) - \sin(2\theta)$; $y = r \sin(\theta) = 2 \sin(\theta) - 2 \sin^2(\theta)$. Hence $\frac{dx}{d\theta} = -2 \sin(\theta) - 2 \cos(2\theta)$; $\frac{dy}{d\theta} = 2 \cos(\theta) - 2 \sin(2\theta)$. Hence $\frac{dy}{dx} = \frac{2 \cos(\theta) - 2 \sin(2\theta)}{-2 \sin(\theta) - 2 \cos(2\theta)}$. Plugging in $\theta = \frac{\pi}{4}$ gives $\sqrt{2} - 1$.

7. **A** $\lim_{t \rightarrow \infty} \frac{\ln(e^t + 1)}{e^t} = \lim_{t \rightarrow \infty} \frac{\frac{e^t}{e^t + 1}}{e^t} = \lim_{t \rightarrow \infty} \frac{1}{e^t + 1} = 0$.

8. **D** For $\ln(x)$, it can be seen that $f^{(n)}(x) = \frac{(-1)^{n+1}(n-1)!}{x^n}$ so $\sum_{n=1}^{\infty} \frac{1}{f^{(n)}(x)} = \sum_{n=1}^{\infty} \frac{(-1)(-x)^n}{(n-1)!} =$

$$\sum_{n=0}^{\infty} \frac{(-1)(-x)^{n+1}}{n!} = x \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = x(e^{-x}).$$

9. **E** Use integration by parts to obtain $\int \frac{\ln(x)}{\sqrt{x}} dx = 2\sqrt{x} \ln(x) - \int \frac{2}{\sqrt{x}} dx = 2\sqrt{x}(\ln(x) - 2)$.

Plugging in the limits, we get $4e^2$.

10. **C** If we multiply top and bottom by $\sqrt{1 - \sin(x)}$, we can rewrite this integral as

$\int_0^{\frac{\pi}{6}} \sqrt{\frac{1 - \sin^2(x)}{(1 - \sin(x))^2}} dx = \int_0^{\frac{\pi}{6}} \frac{\cos(x)}{1 - \sin(x)} dx$. Let $u = 1 - \sin(x)$; then $du = -\cos(x)$ and this evaluates nicely to $-\ln|1 - \sin(x)|$. Plugging in the bounds, we get $\ln(2)$.

11. **D** If $f(x)$ is equal to its Maclaurin series, then its Maclaurin series must also be an even function. Hence every odd power must have a zero coefficient. Thus I is true. By symmetry, II is true (one can verify by making the substitution $u = -x$). III is true; while the "counterexample" $f(x) = c$ may come up in disputes, note that the zero function is by definition both odd and even.

12. **C** Since there's 3 petals, the bounds are $0 \leq \theta \leq \frac{2\pi}{3}$. Hence we get

$$\int_0^{\frac{2\pi}{3}} \frac{r^2}{2} d\theta = \int_0^{\frac{2\pi}{3}} 2 \cos^2(3\theta) d\theta = \int_0^{\frac{2\pi}{3}} (1 + \cos(6\theta)) d\theta = \theta + \frac{1}{6} \sin(6\theta) \text{ evaluated at the bounds, giving } \frac{2\pi}{3}.$$

13. **B** The area bound by the curve is $\int_0^{\frac{\pi}{3}} y dx$; $y = \sin(t)$, $dx = \sec^2(t) dt$; hence, this is

$$\int_0^{\frac{\pi}{3}} \frac{\sin(t)}{\cos^2(t)} dt = \sec\left(\frac{\pi}{3}\right) - \sec(0) = 1.$$

14. **B** The volume bound by the curve will be $\pi \int_0^{\frac{\pi}{3}} y^2 dx$; y and dx are as in question 13; hence we

get $\pi \int_0^{\frac{\pi}{3}} \frac{\sin^2(t)}{\cos^2(t)} dt = \pi \int_0^{\frac{\pi}{3}} \tan^2(t) dt = \pi \int_0^{\frac{\pi}{3}} (\sec^2(t) - 1) dt = \tan(t) - t$ evaluated at the limits, which gives $\frac{3\pi\sqrt{3} - \pi^2}{3}$.

15. **A** $\sqrt{1 - x^2} \frac{dy}{dx} = y^2 + 1 \Rightarrow \frac{dy}{y^2 + 1} = \frac{dx}{\sqrt{1 - x^2}} \rightarrow \arctan(y) = \arcsin(x) + C \rightarrow y = \tan(\arcsin(x) + C)$.

Plug in $(0, 0)$ to get $C = 0$ then plug in $x = \frac{1}{2}$ to get $\frac{\sqrt{3}}{3}$.

16. **E** There's a vertical asymptote at $x = 1$. Integral diverges.

17. **D** Theorem of Pappus: $V = 2\pi rA$, where A is the area of the triangle in question and r is the distance from the centroid to the origin (since the origin lies on the line that the triangle is being rotated about). Hence, to minimize V , we want to minimize $2\pi \frac{ab}{2} \sqrt{(a/3)^2 + (b/3)^2} = \frac{\pi}{3} \left(\frac{1}{b\sqrt{b}}\right)(b) \sqrt{\frac{1}{b^3} + b^2} = \frac{\pi}{3} \sqrt{\frac{1}{b^4} + b}$. It suffices to minimize what's inside the square root and so $1 - \frac{4}{b^5} = 0 \rightarrow b = \sqrt[5]{4}$.

18. **A** $\frac{dy}{dt} = \frac{1}{t}; \frac{dx}{dt} = 2t \rightarrow \frac{dy}{dx} = \frac{1}{2t^2}; \frac{d(\frac{dy}{dx})}{dt} = \frac{-1}{t^3} \rightarrow \frac{d^2y}{dx^2} = -\frac{1}{2t^4}$

19. **B** Making the substitution $u = -x^2$ and integrating, this becomes $2\pi \times -\frac{1}{2}e^{-x^2}$; evaluating at the limits gives $2\pi \times \frac{1}{2} = \pi$.

20. **C** Transforming this sum into an integral gives $\int_0^2 x^4 dx = 2^5/5 = 32/5$.

21. **C** Let $u = x^2$. Then our integral becomes $\int_0^{\frac{1}{2}} \frac{2}{1-u^2} du = \int_0^{\frac{1}{2}} \left(\frac{1}{1-u} + \frac{1}{1+u} \right) du = \ln \left| \frac{1+u}{1-u} \right|$ evaluated at the limits; this is $\ln(3)$.

22. **E** The power series for $\ln(x+1)$ has radius of convergence 1. The series diverges for $x = 2$.

23. **B** Use the Ratio Test to determine the radius: $\lim_{k \rightarrow \infty} \frac{(k+1)^{k+1} x^{k+1}}{(k+1)!} \times \frac{k!}{x^k k^k} = \lim_{k \rightarrow \infty} \frac{x(k+1)^{k+1}}{(k+1)k^k} = \lim_{k \rightarrow \infty} \frac{x(k+1)^k}{k^k} = x \lim_{k \rightarrow \infty} \left(1 + \frac{1}{k}\right)^k = ex$. Since we need $|ex| < 1$, $\frac{1}{e}$ is our ROC.

24. **B** This takes integration by parts and a clever u -substitution. We first integrate by parts with $u = \sqrt{x}$, $dv = e^{-x} dx$ to obtain $-e^{-x} \sqrt{x} + \int_0^\infty \frac{e^{-x}}{2\sqrt{x}} dx$. Note that the first part (that is, $-e^{-x} \sqrt{x}$) goes to zero at both ∞ and 0, and so we're left with $\int_0^\infty \frac{e^{-x}}{2\sqrt{x}} dx$. Now, let $u = \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}} dx$. Then this

integral becomes $\int_0^\infty e^{-u^2} du$. Now we use the hint: since e^{-u^2} is an even function,

$$\int_0^\infty e^{-u^2} du = \frac{1}{2} \int_{-\infty}^\infty e^{-u^2} du = \frac{1}{2} \sqrt{\pi}.$$

25. **A** $\Theta(x) = \frac{1}{1-x} \rightarrow \int_0^{\frac{1}{2}} \Theta(x) dx = -\ln\left(1 - \frac{1}{2}\right) + \ln(1) = \ln(2)$.

26. **D** Definition.

27. **A** This is the same thing as saying $\frac{\mu(x-2)(x-1) + \alpha x(x-1) + \theta x(x-2)}{x(x-1)(x-2)} = \frac{1}{x(x-1)(x-2)}$. Plugging in $x = 2$, $x = 1$, and $x = 0$, we can isolate μ , θ , and α respectively to get $\mu = \alpha = \frac{1}{2}$ and $\theta = -1$. Product $-\frac{1}{4}$.

28. **C** $\pi \int_1^\infty \frac{1}{x^{\frac{3}{2}}} dx = \frac{-2\pi}{\sqrt{x}}$; plugging in the bounds gives 2π .

29. **E** Be careful! Note that $y \neq 3x^2$. y is actually a constant 27 for all x . Hence the average value of y over this interval is 27.

30. **C** Substitute $x = 2 \sin(\theta)$ and evaluate. We wind up with $\arcsin\left(\frac{x}{2}\right)$, evaluated at 2 and 1, to give $\frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$.