For all questions, answer E: "NOTA" should be chosen only if none of the given answers is correct.
For this test, “a square” is taken to be the square of an integer, and “a cube” is taken to be the cube of a positive integer, unless otherwise noted.

1. Which of the following is NOT a pythagorean triple?

   A: (95472, 237154, 255650) B: (54288, 130834, 141650)

   C: (81079, 411000, 418921) D: (115446, 655928, 666012) E: NOTA

2. Suppose a is an integer, and $a!$ is divisible by 10^5 but not 10^6. Solve for x:

   $0 = x^3 + ax + 3$

   A: 0 B: -1

   C: a D: 2, -1 E: NOTA

3. How many positive proper divisors does 224 have?

   A: 10 B: 12

   C: 11 D: 9 E: NOTA

4. If we know that for some integer k, $k \equiv 3 \pmod 4$ and $k \equiv 2 \pmod 7$, what can we say about k?

   A: k is odd B: k cannot be a perfect square

   C: $k \equiv 3 \pmod{28}$ D: Both A and B E: NOTA

5. Find x in a solution (x, y) to the Diophantine equation $37x + 11y = 3$

   A: -9 B: -2

   C: 3 D: 2 E: NOTA

   For questions 6 and 7, let φ(n) represent the Euler totient function; φ(n) = the number of integers less than n which are relatively prime to n.

6. Evaluate φ(36)

   A: 12 B: 35

   C: 15 D: 0 E: NOTA

7. A nontotient is a positive integer n for which the formula $\varphi(x) = n$ has no solution for x. How many of the following are nontotients? 6, 12, 14, 18, 22, 24, 26

A: 1 B: 2
C: 3 D: 4 E: NOTA

8. Which of these sets is NOT *known* to be infinite?

A: The set of all prime numbers
B: The set of Pythagorean triples
C: The set of all primes p such that $p+2$ is prime
D: The set of constructible regular polygons (by compass and straightedge)
E: NOTA

9. Find the sum of the prime divisors of 15147

A: 37 B: 190
C: 40 D: 31 E: NOTA

10. If you are told that k is a positive integer, and that k^2 has 6 proper divisors, what could be true about k?

A: k is prime B: k is a perfect square
C: k is a cube of a prime D: k has 4 proper divisors E: NOTA

11. Which of the following numbers can be expressed as the sum of two cubes in two different ways?

A: 8 B: 1729
C: 72 D: 1216 E: NOTA

12. Find x satisfying the congruence $11^x \equiv 1 \pmod{23}$

A: 22 B: 13
C: 4 D: 16 E: NOTA

13. Let M be the least common multiple of 214 and 191, and let N be their greatest common divisor. What is M – N?

A: 40875 B: 40874

C: 40872 D: 40873 E: NOTA

The next three questions involve partitions. A partition of a natural number is a set of natural numbers which sum to the original number, with order not important. For example, the partitions of 3 are {3}, {1,1,1}, and {1,2}

14. How many partitions are there for the number 6?

A: 10 B: 9

C: 8 D: 7 E: NOTA

15. Which of the following statements are true for any natural number $n \geq 2$?

I. $n+1$ divides $n!$ II. $n+1$ has more partitions than n does

III. There is a prime number between n and $2n$

A: I only B: I and II

C: II only D: I, II and III E: NOTA

16. Let x be the number of partitions of the integer 5, and y be the number of partitions of the integer 4. Find the smallest positive integer z such that $z \equiv x \pmod{y}$

A: 1 B: 2

C: 3 D: 4 E: NOTA

17. Given the following congruences: $x \equiv 2 \pmod{7}$, $x \equiv 1 \pmod{11}$, $x \equiv 4 \pmod{13}$

If the solution is a congruence in the form $x \equiv m \pmod{n}$ where $0 \leq m \leq n$, what is the nearest integer to the ratio n / m ?

A: 0 B: 4

C: 5 D: 2 E: NOTA

18. Find the smallest positive integer x such that $x \equiv 13^{31} \pmod{31}$

A: 0 B: 1

C: 13 D: 26 E: NOTA

19. Evaluate (A + B)(C – A) using the following information:

A = gcd(84, 36) B = lcm(14, 98) C = 5! - 5 D = 81 (mod 7)

A: 11330 B: 11220

C: 12210 D: 11880 E: NOTA

Questions 21, 22 and 23 concern algebraic numbers. An algebraic number is a number that can be the root of a polynomial with integer coefficients $a_0 + a_1x + a_2x^2 + \ldots + a_nx^n = 0$.

20. Which of the following numbers is algebraic?

A: e B: ln 2

C: φ (the golden ratio) D: π E: NOTA

21. Suppose x, y and z are non-zero algebraic numbers. Which of the following must also be algebraic?

I. $x + y$ II. x^y

III. $(x-y)*z$ IV. $\log(z) + x^2$

A: I only B: I and II

C: I and III D: I, II and III E: NOTA

22. Suppose you are told that some number α can be expressed as the solution to a polynomial with rational coefficients. What can you say about α?

A: α is algebraic B: α is not algebraic

C: α may be algebraic D: α is algebraic only if $a_0 = 0$ E: NOTA

23. Which of the following numbers is divisible by 9?

A: 98612347 B: 98327549806 C: 2456877 D: 547948 E: NOTA

24. Convert the number 8741 into base 8 (also known as octal)

A: 2145 B: 21045

C: 71045 D: 41045 E: NOTA

25. What is the probability that a randomly selected factor of 135 is odd?

A: 1/5 B: 3/5

C: 5/12 D: 27/35 E: NOTA

26. Sven is walking up a numbered staircase. The first step is labeled with a 1, the next with a 2, and so on for all 100 steps. Sven decides that every time he stands on a prime number, he will skip the next step. How many steps will Sven stand on?

A: 24 B: 25

C: 75 D: 76 E: NOTA

27. Suppose Sven decides to walk down the staircase, but this time when he stands on a prime number n, he skips the next $\sqrt{n}$ steps, rounded down to the nearest integer (assume he can do this safely and accurately, without falling down the stairs). How many prime-numbered steps does Sven touch on the way down?

A: 11 B: 12

C: 14 D: 25 E: NOTA

28. Which of the following sets is closed under exponentiation; that is, for x and y in the set, x^y and y^x are always members of the set?

I. Real Numbers II. Positive Integers

III. Rational Numbers IV. {0, 1, -1}

A: I and II B: I, II, III and IV

C: I and III D: I, II and IV E: NOTA

29. The Fibonacci sequence is defined recursively as F(0) = 0, F(1) = 1, and F(n) = F(n-1) + F(n-2) for all n >1. What is the sum of the first 6 Fibonacci numbers?

A: 7
B: 20
C: 13
D: 12
E: NOTA

30. A complex number is in the form $a + bi$, where a and b are real numbers, and $i^2 = -1$. Gaussian integers (denoted as $\mathbf{Z}[i]$) are complex numbers $a + bi$, where a and b are integers.

What property of real integers is not true for Gaussian integers?

A: Commutativity of Multiplication
B: Unique Factorization
C: Associativity of Multiplication
D: Totally Ordered
E: NOTA