Answers:

0. 0
1. $\sin 2x$
2. $-1006-1006i$
3. $3/7$
4. 6
5. 9
6. 6000π
7. $2\sqrt{2}, \frac{1}{2}$
8. $(-\infty,-1)\cup\left(-\frac{2}{3},1\right)\cup(1,3)$
9. $5\pi/3$
10. 108
11. 7
12. $1/624$
13. 11
14. 1001

Solutions:

0. Since $\cos 90^\circ = 0$ is part of the product, the entire product is 0.

1. $$\frac{2\cos 2x}{\cot x - \tan x} = \frac{2\left(\cos^2 x - \sin^2 x\right)}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}} = \frac{2\left(\cos^2 x - \sin^2 x\right)}{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}} = 2\sin x \cos x = \sin 2x$$

2. $\sum_{n=1}^{2011} ni^n = \sum_{n=0}^{2011} ni^n$, and each cycle of 4 terms (0~3, 4~7, etc.) sums to $0 + i - 2 - 3i$ $= -2 - 2i$, so because there are 503 such cycles, the sum is $503(-2-2i) =$ $-1006 - 1006i$

3. Let $p =$ the probability of flipping a tail, and let $q =$ the probability of flipping a head on a single flip. The probability of flipping exactly 4 heads is $15p^2q^4$, and the probability of flipping exactly 3 tails is $20p^3q^3$. Setting these equal and dividing out the common p's and q's, $15q = 20p \Rightarrow 3q = 4p$. Since $p + q = 1$, $p = 3/7$ $\left(\text{and } q = 4/7\right)$.

4. $$\lim_{x\to 2} \frac{x-2}{\sqrt{x+7}-3} = \lim_{x\to 2} \frac{(x-2)\left(\sqrt{x+7}+3\right)}{\left(\sqrt{x+7}-3\right)\left(\sqrt{x+7}+3\right)} = \lim_{x\to 2} \frac{(x-2)\left(\sqrt{x+7}+3\right)}{x-2} = \sqrt{2+7}+3 = 6$$

5. $x^2 + 4x + y^2 + 8y = 12 = 0 \Rightarrow (x+2)^2 + (y+4)^2 = 8$, so the center of the circle is at the point $(-2,-4)$, and the distance from the center to the point $(3,4)$ is $\sqrt{(3+2)^2 + (4+4)^2} = \sqrt{89}$. The radius of the circle is $\sqrt{8} = 2\sqrt{2}$, and the radius and the tangent segment are the two legs of a right triangle whose hypotenuse is the segment between the center and the point. Thus, the length of the tangent segment is $\sqrt{89-8} = \sqrt{81} = 9$

6. The mule can roam freely for $2/3$ of a disk with radius 90. Going to the left, the mule can roam an additional $1/6$ of a disk with radius 30. Going to the right, the mule can roam an additional $1/2$ of a disk with radius 30. The total roaming area is thus $\frac{2}{3}\pi(90)^2 + \frac{1}{6}\pi(30)^2 + \frac{1}{2}\pi(30)^2 = 5400\pi + 150\pi + 450\pi = 6000\pi$.

7. $\log_2 x = 1 - \log_2 x + 3\log_x 2 \Rightarrow 2\log_2 x - 1 - 3\log_x 2 = 0 \Rightarrow 2(\log_2 x)^2 - \log_2 x - 3 = 0 \Rightarrow$ $(2\log_2 x - 3)(\log_2 x + 1) = 0 \Rightarrow \log_2 x = \frac{3}{2}$ or $\log_2 x = -1 \Rightarrow x = 2\sqrt{2}$ or $x = \frac{1}{2}$

8. $3x^5 - 10x^4 - 2x^3 + 16x^2 - x - 6 < 0 \Rightarrow (3x+2)(x-3)(x+1)(x-1)^2 < 0$, and the expression is negative on the intervals $(-\infty,-1) \cup \left(-\frac{2}{3},1\right) \cup (1,3)$

9. $\left(\cos 2x + \sqrt{3}\sin 2x\right)^2 = 2 \Rightarrow \left(2\cos\left(\frac{\pi}{3} - 2x\right)\right)^2 = 2 \Rightarrow \cos\left(\frac{\pi}{3} - 2x\right) = \pm\frac{\sqrt{2}}{2} \Rightarrow$ $\frac{\pi}{3} - 2x = \frac{\pi}{4} + \frac{\pi}{2}k \Rightarrow -2x = -\frac{\pi}{12} + \frac{\pi}{2}k \Rightarrow x = \frac{\pi}{24} + \frac{\pi}{4}k$. Since $0 < x < \pi$, $x = \frac{\pi}{24}$, $\frac{7\pi}{24}$, $\frac{13\pi}{24}$, or $\frac{19\pi}{24}$, and the sum of these solutions is $\frac{\pi}{24} + \frac{7\pi}{24} + \frac{13\pi}{24} + \frac{19\pi}{24} = \frac{40\pi}{24} = \frac{5\pi}{3}$

10. $432 = 2^4 \cdot 3^3$, so there are 20 positive integral factors. Since 1, 2, 3, and 4 are all factors, the 17th factor is $432/4 = 108$.

11. $\frac{2x^2 + 3x - 2}{x^3 + 3x^2 + 3x + 1} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} \Rightarrow 2x^2 + 3x - 2 = A(x+1)^2 + B(x+1) + C$. Solving this equation, $A = 2$, $B = -1$, and $C = 3$. Therefore, $A - 2B - C = 2 - 2(-1) - (-3)$ $= 2 + 2 + 3 = 7$

12. Both "WIN" and "FAIL" cannot occur simultaneously, so we need just find the sum of the two probabilities. The W in "WIN" can occur in each of the positions 1 through 24, and, for example, when WIN occurs at the beginning of the sequence, there are 23! ways to arrange the other letters. Each of the other positions has the same number of arrangements, so the probability of "WIN" appearing is $\frac{24 \cdot 23!}{26!} = \frac{24!}{26!}$. Likewise, for "FAIL", the probability is $\frac{23 \cdot 22!}{26!} = \frac{23!}{26!}$. The sum of these probabilities is $\frac{24!}{26!} + \frac{23!}{26!} = \frac{23!(24+1)}{26!} = \frac{1}{26 \cdot 24} = \frac{1}{624}$.

13. $|A| = 5(1) - (-3)(2) = 5 + 6 = 11$, so $\left|AA^T A^{-1}\right| = |A|\left|A^T\right|\left|A^{-1}\right| = 11 \cdot 11 \cdot \frac{1}{11} = 11$

14. 2011! contains $\left\lfloor \frac{2011}{2} \right\rfloor + \left\lfloor \frac{2011}{4} \right\rfloor + \left\lfloor \frac{2011}{8} \right\rfloor + \left\lfloor \frac{2011}{16} \right\rfloor + \left\lfloor \frac{2011}{32} \right\rfloor + \left\lfloor \frac{2011}{64} \right\rfloor + \left\lfloor \frac{2011}{128} \right\rfloor +$ $\left\lfloor \frac{2011}{256} \right\rfloor + \left\lfloor \frac{2011}{512} \right\rfloor + \left\lfloor \frac{2011}{1024} \right\rfloor = 1005 + 502 + 251 + 125 + 62 + 31 + 15 + 7 + 3 + 1 = 2002$ factors of 2 and $\left\lfloor \frac{2011}{3} \right\rfloor + \left\lfloor \frac{2011}{9} \right\rfloor + \left\lfloor \frac{2011}{27} \right\rfloor + \left\lfloor \frac{2011}{81} \right\rfloor + \left\lfloor \frac{2011}{243} \right\rfloor + \left\lfloor \frac{2011}{729} \right\rfloor = 670 + 223$ $+74 + 24 + 8 + 2 = 1001$ factors of 3. Since each factor of 12 consists of two 2's and one 3, there are 1001 12's in the product. In base 12, each of these would create a 0 at the end, so the number ends in 1001 zeros.