Answers:

0. $y = 2x$
1. $\left[\frac{1-\sqrt{2}}{2}, \frac{1+\sqrt{2}}{2}\right]$
2. -1
3. $9\pi/4$
4. $7/3$
5. $10/3$
6. 30
7. $\ln\left(1+\sqrt{2}\right)$
8. 12
9. $1/16$
10. $\frac{\pi-2}{4}$
11. $\pi + 3\sqrt{3}$
12. e^2
13. $\frac{e^{2x}}{8x+4} + C$
14. $-\frac{\sqrt{3}}{2}$

Solutions:

0. $y' = \cos x + 1 \Rightarrow m = y'\big|_{x=0} = \cos 0 + 1 = 1 + 1 = 2$

Since line is through the point $(0,0)$, tangent is $y = 2x$.

1. $y' = \dfrac{(x^2+1)-(x+1)(2x)}{(x^2+1)^2} = \dfrac{-x^2-2x+1}{(x^2+1)^2} \Rightarrow y' = 0$ when $x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(-1)}}{2(-1)}$

$= \dfrac{2 \pm 2\sqrt{2}}{-2} = -1 \pm \sqrt{2}$. $y' > 0$ when $-1-\sqrt{2} < x < -1+\sqrt{2}$ and $y' < 0$ when

$x < -1-\sqrt{2}$ or $x > -1+\sqrt{2}$, and graph has a horizontal asymptote in both directions

at $y = 0$. When $x = -1-\sqrt{2}$, $y = \dfrac{-1-\sqrt{2}+1}{(-1-\sqrt{2})^2+1} = \dfrac{-\sqrt{2}}{4+2\sqrt{2}} = \dfrac{1-\sqrt{2}}{2}$, and when

$x = -1+\sqrt{2}$, $y = \dfrac{-1+\sqrt{2}+1}{(-1+\sqrt{2})^2+1} = \dfrac{\sqrt{2}}{4-2\sqrt{2}} = \dfrac{1+\sqrt{2}}{2}$. So the range is $\left[\dfrac{1-\sqrt{2}}{2}, \dfrac{1+\sqrt{2}}{2}\right]$.

2. $2xy\dfrac{dy}{dx} + y^2 + \cos x + \dfrac{dy}{dx} = \dfrac{y^2 - 2xy\dfrac{dy}{dx}}{y^4} \Rightarrow 0 + 1 + 1 + \dfrac{dy}{dx}\bigg|_{(0,1)} = \dfrac{1-0}{1} \Rightarrow \dfrac{dy}{dx}\bigg|_{(0,1)} = -1$

3. Integral represents area enclosed by circle $x^2 + y^2 = 9$ in the first quadrant. So the answer is ${}^{9\pi}\!/_{4}$.

4. $\lim\limits_{n\to\infty}\sum\limits_{x=1}^{n}\left(\dfrac{x^2+2n^2}{n^3}\right) = \lim\limits_{n\to\infty}\sum\limits_{x=1}^{n}\left(\left(\dfrac{x}{n}\right)^2 + 2\right)\cdot\dfrac{1}{n} = \int_0^1 (x^2+2)\,dx = \dfrac{1}{3}x^3 + 2x\bigg|_0^1 = \dfrac{1}{3} + 2 - 0 = \dfrac{7}{3}$

5. $\dfrac{6}{x} = \dfrac{15}{x+z} \Rightarrow 6x + 6z = 15x \Rightarrow 6z = 9x \Rightarrow x = \dfrac{2}{3}z \Rightarrow \dfrac{dx}{dt} = \dfrac{2}{3}\dfrac{dz}{dt}$. Since $\dfrac{dz}{dt} = 5$, $\dfrac{dx}{dt} = \dfrac{2}{3}\cdot 5$

$= \dfrac{10}{3}$

6. $g'(x) = \dfrac{1}{f'(g(x))}$, so $(g'(-6))^{-1} = f'(g(-6)) = f'(2) = 6(2)^2 + 6 = 30$

7. $y' = \dfrac{-\sin x}{\cos x} = -\tan x$, so length is $\int_0^{\pi/4}\sqrt{1+\tan^2 x}\,dx = \int_0^{\pi/4}\sec x\,dx = \ln\left|\sec x + \tan x\right|\Big|_0^{\pi/4}$

$= \ln\left(1+\sqrt{2}\right)$

8. The side of the rectangle that is parallel to the one on the hypotenuse creates a smaller right triangle that shares the right angle in the original triangle, so let the sides of that triangle, which are in the same 3:4:5 ratio, be x, $\frac{4}{3}x$, and $\frac{5}{3}x$. Then the altitude to the hypotenuse in the smaller right triangle is $\frac{4}{5}x$. Since the altitude to the hypotenuse in the larger triangle has length $\frac{24}{5}$, the height of the rectangle is $\frac{24}{5}-\frac{4}{5}x$, and the hypotenuse of the smaller triangle is the other side. So the area is $A=\left(\frac{5}{3}x\right)\left(\frac{24}{5}-\frac{4}{5}x\right)=8x-\frac{4}{3}x^2 \Rightarrow A'=8-\frac{8}{3}x \Rightarrow A'=0$ when $x=3$, and this creates a maximum since the graph of A is a parabola opening downward. Thus the area is $A(3)=8\cdot 3-\frac{4}{3}(3)^2=24-12=12$.

9. Let $h=$ height of water in tank. Then the volume of water is $V=15\cdot\frac{1}{2}\cdot\frac{4}{3}h\cdot h=10h^2$ $\Rightarrow \frac{dV}{dt}=20h\frac{dh}{dt} \Rightarrow 2.5=20\cdot 2\frac{dh}{dt} \Rightarrow \frac{dh}{dt}=\frac{1}{16}$

10. $\int_0^{\pi/4}(\cos x-\sin x)^2\,dx=\int_0^{\pi/4}\left(\cos^2 x+\sin^2 x-2\sin x\cos x\right)dx=\int_0^{\pi/4}(1-\sin 2x)\,dx$

$= x+\frac{1}{2}\cos 2x\Big|_0^{\pi/4}=\frac{\pi}{4}+0-0-\frac{1}{2}=\frac{\pi-2}{4}$

11. $2\left(\frac{1}{2}\int_0^{2\pi/3}(1+2\cos\theta)^2\,d\theta-\frac{1}{2}\int_{2\pi/3}^{\pi}(1+2\cos\theta)^2\,d\theta\right)=\int_0^{2\pi/3}\left(1+4\cos\theta+4\cos^2\theta\right)d\theta$

$-\int_{2\pi/3}^{\pi}\left(1+4\cos\theta+4\cos^2\theta\right)d\theta=\int_0^{2\pi/3}(3+4\cos\theta+2\cos 2\theta)\,d\theta$

$-\int_{2\pi/3}^{\pi}(3+4\cos\theta+2\cos 2\theta)\,d\theta=\left(3\theta+4\sin\theta+\sin 2\theta\Big|_0^{2\pi/3}\right)-\left(3\theta+4\sin\theta+\sin 2\theta\Big|_{2\pi/3}^{\pi}\right)$

$=2\pi+2\sqrt{3}-\frac{\sqrt{3}}{2}-3\pi+2\pi+2\sqrt{3}-\frac{\sqrt{3}}{2}=\pi+3\sqrt{3}$

12. $\lim_{x\to 0}\csc x\cdot\ln(1+2x)=\lim_{x\to 0}\frac{\ln(1+2x)}{\sin x}=\lim_{x\to 0}\frac{\frac{2}{1+2x}}{\cos x}=\frac{2}{1}=2 \Rightarrow \lim_{x\to 0}(1+2x)^{\csc x}=e^2$

13. $u = xe^{2x}$, $dv = (2x+1)^{-2}\,dx \Rightarrow du = (2x+1)e^{2x}dx$, $v = -\frac{1}{2}(2x+1)^{-1}$

$$\int \frac{xe^{2x}}{(2x+1)^2}dx = -\frac{xe^{2x}}{2(2x+1)} + \frac{1}{2}\int e^{2x}dx = -\frac{xe^{2x}}{2(2x+1)} + \frac{1}{4}e^{2x} + C = \frac{-2xe^{2x} + 2xe^{2x} + e^{2x}}{4(2x+1)} + C$$

$$= \frac{e^{2x}}{8x+4} + C$$

14. $x(t) = \frac{1}{2}\sin 2t \Rightarrow x'(t) = \cos 2t \Rightarrow x''(t) = -2\sin 2t \Rightarrow x''(t) = 1$ when $\sin 2t = -\frac{1}{2}$

$$\Rightarrow 2t = \frac{7\pi}{6} \Rightarrow x'(t) = \cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$$