Answers:

0. 252°
1. $\frac{2\sqrt{2}}{3}$
2. $\frac{1}{10}$
3. 3
4. $2+3i$
5. 80
6. $1/52$
7. 18
8. 4π
9. $-1/3$ and $7/3$
10. $1/18$
11. $\sqrt{337}$
12. $6\sqrt{3}-6$
13. $\frac{20+6\sqrt{10}}{5}$
14. 3

Solutions:

0. Let x be the larger arc. Then $\frac{x-(360-x)}{2}=72 \Rightarrow 2x-360=144 \Rightarrow 2x=504$

$x=252^\circ$

1. The circle encloses $\frac{1}{3}$ of the area enclosed by the ellipse, so $3r=3b=a$. Therefore,

$c=b\sqrt{3^2-1^2}=2\sqrt{2}b=\frac{2\sqrt{2}}{3}a \Rightarrow \frac{c}{a}=\frac{2\sqrt{2}}{3}$

2. $(\log x)^2=\log\left(x^{\log x}\right)=\log(100x)=\log x+\log 100=\log x+2$

$\Rightarrow (\log x)^2-\log x-2=0 \Rightarrow (\log x-2)(\log x+1)=0 \Rightarrow \log x=2 \text{ or } -1$

$\Rightarrow x=100 \text{ or } \frac{1}{10}$

3. Perpendicular bisector of line segment goes through the point $(5,3)$ and has slope $-1 \Rightarrow y=-x+8$. This line intersects $y=2x-1$ at the point $(3,5)$.

4. $2(a+bi)+5(a-bi)=14-9i \Rightarrow 7a=14 \text{ and } -3b=-9 \Rightarrow a=2 \text{ and } b=3 \Rightarrow z=2+3i$

5. $\binom{5}{2}\left(x^2\right)^2(2y)^3=80x^4y^3$

6. $\frac{50!-49!}{51!-2(49!)}=\frac{49(49!)}{(51\cdot 50-2)(49!)}=\frac{49}{2548}=\frac{1}{52}$

7. $0\le 10-x\le 2 \Rightarrow 8\le x\le 10 \Rightarrow a+b=8+10=18$

8. This is the area enclosed by the circle centered at the origin with radius 4 that is outside the ellipse centered at the origin with major axis of length 8 and minor axis of length 6. Therefore, the area is $\pi\left(4^2-4\cdot 3\right)=4\pi$.

9. If $x\le 0$, the equation is $-x+(1-x)+(2-x)=4 \Rightarrow 3-3x=4 \Rightarrow 3x=-1 \Rightarrow x=-\frac{1}{3}$. If $0<x\le 1$, the equation is $x+(1-x)+(2-x)=4 \Rightarrow 3-x=4 \Rightarrow x=-1$. If $1<x\le 2$, the equation is $x+(x-1)+(2-x)=4 \Rightarrow x+1=4 \Rightarrow x=3$. If $x>2$, the equation is

$x+(x-1)+(x-2)=4 \Rightarrow 3x-3=4 \Rightarrow 3x=7 \Rightarrow x=\frac{7}{3}$. The only solutions which are in the appropriate ranges are $-\frac{1}{3}$ and $\frac{7}{3}$.

10. There are $6!=720$ possible distributions of checks into the envelopes. There are $\binom{6}{3}=20$ different selections for correct checks, but there are only 2 different ways for the incorrect checks to be distributed. Thus, there are 40 different possible distributions, giving the probability as ${}^{40}\!/_{720}={}^{1}\!/_{18}$.

11. $c^2=8^2+13^2-2(8)(13)\cos 120^\circ=64+169+104=337 \Rightarrow c=\sqrt{337}$

12. $288\pi=\frac{4}{3}\pi r^3 \Rightarrow r^3=216 \Rightarrow r=6$. $3=\frac{4\pi R^2}{4\pi r^2}=\frac{R^2}{r^2} \Rightarrow R^2=3\cdot 6^2=108$, so $R=6\sqrt{3}$. Therefore, the paint is of thickness $6\sqrt{3}-6$.

13. Since the slope is 3, for each increase of 3 in the y-coordinate, the armadillo walks $\sqrt{10}$ up the line. Since the armadillo walks 4 units, the increase in the y-coordinate is $\frac{4\cdot 3}{\sqrt{10}}=\frac{6\sqrt{10}}{5}$, making the new y-coordinate $\frac{20+6\sqrt{10}}{5}$.

14. $\frac{(8000000)^{\frac{2}{3}}\sqrt{0.0009}}{(20)^2}=\frac{200^2(.03)}{20^2}=100(.03)=3$