Answers:

1. B
2. B
3. C
4. A
5. D
6. D
7. B
8. C
9. B
10. A
11. D
12. E
13. C
14. B
15. D
16. A
17. D
18. C
19. B
20. B
21. A
22. D
23. C
24. D
25. D
26. A
27. A
28. C
29. B
30. B

Solutions:

1. $$2\cos\frac{11\pi}{12}=2\cos\frac{{}^{11\pi}\!/_{6}}{2}=-2\sqrt{\frac{1+\cos\frac{11\pi}{6}}{2}}=-2\sqrt{\frac{2+\sqrt{3}}{4}}=-\sqrt{2+\sqrt{3}}$$

2. $$\lim_{h\to 0}\frac{(1+h)^3-2(1+h)^2+1-1^3+2(1)^2-1}{h}=\lim_{h\to 0}\frac{3h+3h^2+h^3-4h-2h^2}{h}$$
$$=\lim_{h\to 0}\frac{h\left(3+3h+h^2-4-2h\right)}{h}=3+0+0-4-0=-1$$

3. $$-5=\begin{vmatrix} y & x & -y \\ 5 & 1 & -2 \\ x & y & x \end{vmatrix}=xy-2x^2-5y^2+xy+2y^2-5x^2\Rightarrow 7x^2-2xy+3y^2-5=0$$
$$B^2-4AC=(-2)^2-4\cdot 7\cdot 3=4-84=-80<0,\ B\neq 0\Rightarrow \text{ellipse}$$

4. $$\tan 2\theta=\frac{B}{A-C}=\frac{-2}{7-3}=-\frac{1}{2}\Rightarrow \sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$$

5. $$f^6(0)\cdot g^4(2)=f^5(2)\cdot g^3(0)=f^4(1)\cdot g^2(-3)=f^3(-2)\cdot g(3)=f^2(-1)\cdot -1=-f(3)=3$$

6. $$\frac{f^{-1}(3)-f^{-1}(0)}{3-0}=\frac{-1-(-3)}{3}=\frac{2}{3}$$

7. The area of the base is
$$\begin{array}{r|rr|r} & 2 & 0 & \\ 0 & -1 & 3 & 6 \\ 0 & 0 & 7 & -7 \\ 21 & 3 & 14 & 0 \\ 56 & 4 & 11 & 33 \\ 55 & 5 & 5 & 20 \\ 10 & 2 & 0 & 0 \\ 142 & & & 52 \end{array}\Rightarrow A=\frac{1}{2}\left|142-52\right|=45,\text{ and the height is } 9$$
So the volume is $V=\frac{1}{3}(45)(9)=135$

8. $$\left(\left(4\operatorname{cis}25^\circ\right)\left(\operatorname{cis}20^\circ\right)\right)^3=\left(4\operatorname{cis}45^\circ\right)^3=64\operatorname{cis}135^\circ=64\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)=-32\sqrt{2}+32i\sqrt{2}$$

9. $h=\frac{2A}{b}=\frac{2\sqrt{24\cdot 3\cdot 14\cdot 7}}{21}=\frac{2\cdot 84}{21}=8$

10. Let $\alpha=\sin^{-1}\left(\frac{7}{25}\right)$ and $\beta=\tan^{-1}\left(\frac{15}{8}\right)$. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$=\left(\frac{24}{25}\right)\left(\frac{8}{17}\right)-\left(\frac{7}{25}\right)\left(\frac{15}{17}\right)=\frac{192-105}{425}=\frac{87}{425}$

11. There are 8 possible birth orders, only 7 of which have at least one girl (BBB doesn't). There is only one order with three girls (GGG), so the answer is $1/7$.

12. By Remainder Theorem, remainder is $f(-1)=-1-9+4+17+3+1-5+7=17$.

13. Jean-Robert Argand (Argand plane)

14. $0=4\sin^4\theta+2\sin^3\theta-4\sin^2\theta-\sin\theta+1=(\sin\theta+1)(2\sin\theta-1)(2\sin^2\theta-1)$

So $\sin\theta=-1\Rightarrow\theta=-\frac{\pi}{2}$; or $\sin\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{6},\frac{5\pi}{6}$; or $\sin\theta=\pm\frac{\sqrt{2}}{2}\Rightarrow\theta=\pm\frac{\pi}{4},\pm\frac{3\pi}{4}$

$-\frac{\pi}{2}+\frac{\pi}{6}+\frac{5\pi}{6}+\frac{\pi}{4}-\frac{\pi}{4}+\frac{3\pi}{4}-\frac{3\pi}{4}=\frac{\pi}{2}$

15. $x=\left(\frac{y+1}{2}\right)^2+\frac{y+1}{2}=\frac{1}{4}y^2+y+\frac{3}{4}\Rightarrow 4x=y^2+4y+3\Rightarrow x=\frac{1}{4}(y+2)^2-\frac{1}{4}$

The vertex is at the point $\left(-\frac{1}{4},-2\right)$

16. $S=\frac{3/4}{1-3/4}+\frac{1/2}{1-1/2}=\frac{3/4}{1/4}+\frac{1/2}{1/2}=3+1=4$

17. $f(x)=\log_x\left(6x^3-31x^2+34x+15\right)=\log_x(2x-5)(3x+1)(x-3)\Rightarrow x\neq 1$, $x>0$, and

$(2x-5)(3x+1)(x-3)>0$ when $x\in\left(-\frac{1}{3},\frac{5}{2}\right)\cup(3,\infty)$, so the answer is

$(0,1)\cup\left(1,\frac{5}{2}\right)\cup(3,\infty)$

18. $(23)^{23}(16)^{16}-(8)^8(12)^{12}$ ends in $7\cdot 6-6\cdot 6=42-36=6$

19. $2011_9=2(9)^3+9+1=1468=4\cdot 367$ and $4022_5=4(5)^3+2(5)+2=512=4\cdot 128$. The greatest common factor of the two numbers is 4, which is the same as 100_2.

20. $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$, so I is false; $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ is a property, so II is true; $(AB)^T=B^TA^T$ is a property of matrices, so III is true; $\vec{a}\cdot\vec{a}$ is a real number, while $\left(\vec{a}\right)^2$ has no value, so IV is false; $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}\times\vec{b}\right)\times\vec{c}$ is a property of vectors, so V is true; thus the answer is B

21. $\binom{10}{3}$ chooses the toppings, $(8-1)!$ chooses the crusts for each slice, so the total number of pizzas is $\binom{10}{3}\cdot(8-1)!=\frac{10!}{3!7!}\cdot 7!=\frac{10!}{3!}$

22. $\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 3 & 2 & -1\\ -1 & 4 & -1\end{vmatrix}=2\vec{i}+4\vec{j}+14\vec{k}$ and $\vec{c}\times\vec{d}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 7 & -2 & 5\\ -1 & 1 & 1\end{vmatrix}=-7\vec{i}-12\vec{j}+5\vec{k}$, so

$$\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & 4 & 14\\ -7 & -12 & 5\end{vmatrix}=188\vec{i}-108\vec{j}+4\vec{k}$$

23. $\frac{3-5i}{i+1}\cdot\frac{1-i}{1-i}-\frac{7-i}{2-3i}\cdot\frac{2+3i}{2+3i}=\frac{-2-8i}{2}-\frac{17+19i}{13}=\frac{-13-52i}{13}-\frac{17+19i}{13}=-\frac{30}{13}-\frac{71}{13}i$

24. $(x+2)\ln 2=(2x+1)\ln 3\Rightarrow 2\ln 2-\ln 3=x(2\ln 3-\ln 2)\Rightarrow x=\frac{\ln 4/3}{\ln 9/2}=\log_{4.5}4/3\Rightarrow$

$$\frac{9}{2b}=\frac{9}{2\left(4/3\right)}=\frac{9}{8/3}=\frac{27}{8}$$

25. Sum of reciprocals $=-\frac{-9}{-3}=-3$, and $2S_2-13\left(13/2\right)+7\cdot 2=0\Rightarrow S_2=\frac{141}{4}\Rightarrow$

$$\frac{141}{4}-(-3)=\frac{141}{4}+\frac{12}{4}=\frac{153}{4}$$

26. $x=\frac{6!}{2!2!}=\frac{720}{4}=180$, so $\log x=\log 2+2\log 3+\log 10=a+2b+1$

27. $9x^2-16y^2+18x+64y-199=0 \Rightarrow 9(x+1)^2-16(y-2)^2=199+9-64=144 \Rightarrow$

$\frac{(x+1)^2}{16}-\frac{(y-2)^2}{9}=1$, so a focus is at $(-1+5,2)=(4,2)$

One asymptote is $y-2=\frac{3}{4}(x+1) \Rightarrow 3x-4y+11=0$, so the distance is

$\frac{|3(4)-4(2)+11|}{\sqrt{3^2+4^2}}=\frac{15}{5}=3$

28. $x^2=2^2+4^2-2\cdot 2\cdot 4\cos 135^\circ=20+8\sqrt{2}=4\left(5+2\sqrt{2}\right) \Rightarrow x=2\sqrt{5+2\sqrt{2}}$

29. Since the fourth row is twice the first row, but the other rows are not linear combinations of each other, the rank is 3. The trace is the sum of the entries on the main diagonal, which is $1-2+4+6=9$. The sum of the rank and trace is 3+9=12.

30. $0 \le \frac{3x-2}{2x+1}-\frac{x-1}{x+3}=\frac{(3x-2)(x+3)-(x-1)(2x+1)}{(2x+1)(x+3)}=\frac{x^2+8x-5}{(2x+1)(x+3)}$

$=\frac{\left(x+4+\sqrt{21}\right)\left(x+4-\sqrt{21}\right)}{(2x+1)(x+3)}$, and that expression is nonnegative when

$x \in \left(-\infty,-4-\sqrt{21}\right] \cup \left(-3,-\frac{1}{2}\right) \cup \left[-4+\sqrt{21},\infty\right)$