For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. In calculus, one learns of function representations that are infinite series called "power series". For instance, $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \ldots$. Which of the following is the equivalent expression of e^x? (Define $x^0 = 1$ for all x)

A) $\sum_{i=0}^{\infty} \frac{x^i}{(i+1)!}$ B) $\sum_{i=0}^{\infty} \frac{x^{i-1}}{i!}$ C) $\sum_{i=1}^{\infty} \frac{x^{i-1}}{i!}$ D) $\sum_{i=1}^{\infty} \frac{x^{i-1}}{(i-1)!}$ E) NOTA

2. Refer to question 1: Which of the following is the power series for $\cosh x$, where x is a real number? (Again, define $x^0 = 1$ for all x)

A) $\sum_{i=0}^{\infty} \frac{(-1)^i}{(2i)!} x^{2i}$ B) $\sum_{i=0}^{\infty} \frac{1}{(2i)!} x^{2i}$ C) $\sum_{i=0}^{\infty} \frac{(-1)^i}{(2i+1)!} x^{2i+1}$ D) $\sum_{i=0}^{\infty} \frac{1}{(2i+1)!} x^{2i+1}$ E) NOTA

3. Another important result of power series is $\sum_{i=0}^{\infty} i z^{i-1} = \frac{1}{(1-z)^2}$, where z is a complex number such that $|z| < 1$. What is the value of the series $\sum_{n=1}^{\infty} n\left(\frac{1}{4} - \frac{1}{4}i\right)^{n-1}$?

A) does not converge B) $\frac{8}{25} - \frac{32}{25}i$ C) $\frac{8}{25} + \frac{32}{25}i$ D) $\frac{32}{25} + \frac{8}{25}i$ E) NOTA

4. A result of power series is that $\sum_{i=0}^{\infty} z^i = \frac{1}{1-z}$, where z is a complex number such that $|z| < 1$. Using this, what is the coefficient of the z^2 term of the power series for $\frac{5z+1}{z^2+z-2}$, where $|z| < 1$?

A) $-15/8$ B) $-13/8$ C) $-11/8$ D) $-3/8$ E) NOTA

5. Considering all distinct permutations of the letters in POWERSERIES, what is the probability that a randomly selected permutation begins with POWER?

A) $1/24640$ B) $1/12320$ C) $1/9240$ D) $1/1540$ E) NOTA

6. When the graph of $7x^2-5y^2+20y-23=0$, where $-2\le x\le 2$, is revolved around the line $x=0$, which of the following best represents the surface that is formed?

A) a submarine B) a nuclear cooling tower C) one car headlight D) two car headlights E) NOTA

7. A circular waterwheel with radius 7 meters has a small bucket attached to its outside. The waterwheel begins spinning, and you observed that two seconds later, the bucket is at its greatest height. If the waterwheel rotates at 6 revolutions/minute, and its center is 6 meters above the water, which equation describes the distance d of the bucket above the surface of the water in terms of the number of seconds t after the waterwheel begins spinning?

A) $d=7+6\cos\left(\frac{\pi}{5}(t-2)\right)$ B) $d=7+6\cos\left(\frac{\pi}{10}(t-2)\right)$ C) $d=6+7\cos\left(\frac{\pi}{5}(t-2)\right)$

D) $d=6+7\cos\left(\frac{\pi}{10}(t-2)\right)$ E) NOTA

8. Where defined, which of the following is equivalent to $\csc x$?

A) $\csc x\left(\cos^2 x+\sin x\right)$ B) $\cot x+\csc x$ C) $\csc x\cos^2 x+\sin x$ D) $\cot x\csc x+\sec^2 x$ E) NOTA

9. Let $n>3$ be an integer. Simplify the product $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\ldots\left(1-\frac{1}{n}\right)$.

A) $\frac{1}{n}$ B) $\frac{1}{n+1}$ C) $\frac{2}{n+1}$ D) $\frac{2}{n-1}$ E) NOTA

10. Solve for y: $y^y+2y\left(y^{y-1}\right)=768$

A) 4 B) 3 C) 2 D) -2 E) NOTA

11. Function f satisfies $f(x+y)=f(x)+f(y)$ for all real numbers x and y. If $f(1)=2$, which of the following statements is true?

I) $f(0)=1$ II) $f(2.5)=5$ III) $f(10)=20$

A) III only B) I and II only C) II and III only D) I, II, and III E) NOTA

12. If $0 < x < \pi/4$ and $\cos x + \sin x = 5/4$, find the value of $\cos x - \sin x$.

A) $7/16$ B) $15/16$ C) $25/16$ D) $27/16$ E) NOTA

13. Which of the following dimensions could not be for the figure they describe?

A) quadrilateral with side lengths 2, 3, 4, & 7 ft. B) triangle with side length 5 m
C) triangle with side lengths 6, 7, & 11 in. D) triangle with side lengths 8, 7, & 15 dm
E) NOTA

14. For integers n, k, and r, where $n > k > r \geq 1$, $\binom{n}{k} = \binom{n}{r}$. What is the smallest possible sum of $k + r$?

A) 2 B) 3 C) 4 D) 5 E) NOTA

15. Find the sum: $\sum_{i=1}^{\infty} \frac{i}{5^i}$

A) $2/5$ B) $3/5$ C) $5/16$ D) $9/16$ E) NOTA

16. If x, y, and z are non-zero, consecutive integers with $x < y < z$, which of the following yields a positive odd integer?

A) $x - y - z$ B) $xy + z$ C) $x + y - z$ D) $xz - y$ E) NOTA

17. If $z = \log_{2/3} a$ solves the equation $2\left(4^z\right) + 6^z = 9^z$, find the value of a.

A) $1/4$ B) $1/2$ C) $3/4$ D) $5/4$ E) NOTA

18. Which of the following statements are true?

I) If $a/b = c/d$, then $x^{a/b} = x^{c/d}$, where x is any non-zero integer.

II) $-1 + i\sqrt{3}$ is a cube root of 8.

III) If y is a negative number and z is a rational number, then y^z will be an imaginary number.

A) I & II only B) I & III only C) II & III only D) I, II, & III E) NOTA

19. Find the vector equation of the line of intersection of the planes with equations $x-y-2z=-1$ and $x-3y-3z=-7$.

A) $\vec{r}=\left(2+\frac{3}{\sqrt{14}}d\right)\vec{i}+\left(3-\frac{1}{\sqrt{14}}d\right)\vec{j}+\left(\frac{2}{\sqrt{14}}d\right)\vec{k}$

B) $\vec{r}=\left(2+\frac{3}{\sqrt{14}}d\right)\vec{i}+\left(3+\frac{1}{\sqrt{14}}d\right)\vec{j}+\left(\frac{2}{\sqrt{14}}d\right)\vec{k}$

C) $\vec{r}=\left(2-\frac{3}{\sqrt{14}}d\right)\vec{i}+\left(3-\frac{1}{\sqrt{14}}d\right)\vec{j}+\left(\frac{2}{\sqrt{14}}d\right)\vec{k}$

D) $\vec{r}=\left(2-\frac{3}{\sqrt{14}}d\right)\vec{i}+\left(3+\frac{1}{\sqrt{14}}d\right)\vec{j}+\left(\frac{2}{\sqrt{14}}d\right)\vec{k}$ E) NOTA

20. The fourth roots of $16cis80^\circ$ correspond to four angles in standard position in the plane. What acute angle is formed by the second quadrant angle and the x-axis?

A) 20° B) 40° C) 70° D) 80° E) NOTA

21. If (a,b,c), where $a<b<c$, is the solution to the system $\begin{cases} x+y+z=1 \\ xy+xz+yz=-22 \\ xyz=-40 \end{cases}$, find $a^{c/b}$.

A) 25 B) 9 C) 4 D) -36 E) NOTA

22. If $(x,8)$ is a Cartesian ordered pair on the graph of $r=\tan^2\theta\sec\theta$, then $x=$?

A) -4 B) 2 C) 16 D) 64 E) NOTA

23. Find the sum of the real values of k that make the matrix $\begin{bmatrix} -2 & 4k & 5 \\ 0 & 3 & 2 \\ k & -3 & -k \end{bmatrix}$ singular.

A) $-9/16$ B) 0 C) $9/8$ D) 2 E) NOTA

24. How many solutions exist for $\cos2x-\cos4x=\sqrt{3}\sin3x$ for $0<x<2\pi$?

A) 6 B) 5 C) 4 D) 3 E) NOTA

25. What is the cotangent of the angle that the diagonal of a cube makes with one of its edges?

A) $\sqrt{2}/2$ B) $\sqrt{3}/3$ C) $\sqrt{2}$ D) $\sqrt{3}$ E) NOTA

26. Which of the following statements is true?

I) A graph through the points $(2,109)$, $(3,143)$, $(4,187)$, $(5,241)$, and $(6,305)$ could define a line.

II) A graph through the points $(1,100)$, $(4,64)$, $(7,118)$, $(10,262)$, and $(13,496)$ could define a parabola.

A) I only B) II only C) I & II D) neither I nor II E) NOTA

27. How many of the following statements are true?

I) The equation $7x^2+13xy+3y^2+4x-2y+11=0$ represents a hyperbola.

II) The eccentricity of a circle is always 1.

III) The distance from the center of $2x^2+3y^2-4x+12y=4$ to a focus is 3.

A) 0 B) 1 C) 2 D) 3 E) NOTA

28. If $f(x)=\sqrt{x-15}$ and $g(x)=16-x^2$, find the range of $f\left(g\left(\cos\left(\frac{x-\pi}{2}\right)\right)\right)$.

A) $(-\infty,\infty)$ B) $\left[-1/2,1/2\right]$ C) $\left[0,1/2\right]$ D) $[0,1]$ E) NOTA

29. For $q(x)=\dfrac{2x^5-14x^3}{x^5+4x^4-7x^3-28x^2}$, $y=M$ is the horizontal asymptote, while A and O represent, respectively, the number of vertical and oblique asymptotes $q(x)$ has. Find $M+A+O$.

A) 6 B) 5 C) 3 D) 2 E) NOTA

30. If M and m are the maximum and minimum values, respectively, of $P=5x+9y$, subject to the constraints $x-3y\geq -9$, $2x+5y\leq 15$, $x+y\geq -1$, and $2x-5y\leq 5$, find the value of $M-m$.

A) 43 B) 36 C) 31 D) 24 E) NOTA