This test consists of five relays of six questions each. "TAFTPQITR" stands for "the answer from the previous question in the relay", so if question 3 in a relay references TAFTPQITR, that is the answer from question 2 in that relay.

All answers on this test are integers.

Relay 1

1. Find the sum of the coordinates of the point of intersection of the lines $16x-17y=44$ and $13x+5y=111$.

2. Let $T=$TAFTPQITR. The supplement of an angle θ is 10° more than T times the complement of θ. Find the number of degrees in the measure of θ.

3. Let $T=$TAFTPQITR. What is the maximum number of points of intersection in a plane of T distinct lines?

4. Let $T=$TAFTPQITR. What is the smallest integer n satisfying $n\geq\sqrt{T}$?

5. Let $T=$TAFTPQITR. The only solution in positive integers to the equation $x^2+y^2=T$, where $x<y$, is (a,b). Find the product ab.

6. Let $T=$TAFTPQITR. One chord of a circle has length T, and a second chord of the same circle intersects the first chord. The second chord is divided by the first chord into two segments of length 9 and 10. The first chord is divided by the second chord into two segments. Find the length of the larger of these two segments.

Relay 2

1. A CD has 12 songs, and you have four favorite songs on the CD. If you put the stereo on random play, what is the reciprocal of the probability that the first two songs played are two of your favorites? A song may be played more than once on random play.

2. Let $T=$TAFTPQITR. Find the largest integer x such that the sum of the digits of $x!$ is not divisible by T.

3. Let $T=$TAFTPQITR. The region bounded by the graph of $y=-T|x-1|+3$ and the x-axis has area equal to A. Find the value of $10A$.

4. Let $T =$ TAFTPQITR. Two externally tangent circles have a common external tangent, and the distance between the points of intersection of each circle with the tangent is T. If the smaller circle has a radius of length 3, find the length of the larger circle's radius.

5. Let $T =$ TAFTPQITR. An increasing geometric sequence with first term 1 has T as one of its terms. If every term in the sequence is an integer, find the sum of all possible values of the common ratio of the sequence.

6. Let $T =$ TAFTPQITR. An ellipse has eccentricity $\frac{T+5}{T+7}$ and latus rectum of length $(T-42)^2$. Find the length of the major axis of the ellipse.

Relay 3

1. The only vertical asymptote for the function $f(x)=\frac{6x^2+7x-5}{8x^2-18x+7}$ is $x=a$. Find the value of $4a$.

2. Let $T =$ TAFTPQITR. Find the value of the discriminant of the quadratic equation $3x^2+14x+T=0$.

3. Let $T =$ TAFTPQITR. An isosceles triangle has largest angle of measure T°. How many degrees are in the measure of either of the other two angles?

4. Let $T =$ TAFTPQITR. Find the sum of the positive integral factors of T.

5. Let $T =$ TAFTPQITR. The length of the angle bisector to the shortest side in a triangle with side lengths T, $\frac{2}{3}T$, and $\frac{5}{6}T$ can be written as $\frac{a\sqrt{b}}{c}$, where a and c are relatively prime and b is not divisible by the square of any integer. Find the value of $b\cdot c-a$.

6. Let $T =$ TAFTPQITR. Find the value of $T\cos\left(T-1-\frac{360}{T+1}\right)^\circ$.

Relay 4

1. A circle with radius of length 7 encloses an area of A square units. What is the value of A, rounded to the nearest 10?

2. Let $T =$ TAFTPQITR. A convex $T/10$-gon has a total of how many diagonals?

3. Let $T =$ TAFTPQITR. A secret society with $T/10$ members has a secret handshake that requires the use of three members, rather than just the normal two. If handshakes are performed at their monthly meeting such that each threesome shakes hands exactly once, in how many handshakes is Bob, the charter member of the society, involved?

4. Let $T =$ TAFTPQITR. T is the sum of all integers from 1 to x, inclusive. Find x.

5. Let $T =$ TAFTPQITR. Matrix $B = \begin{bmatrix} 3 & 2 \\ k & 2 \end{bmatrix}$ is such that the matrix product $B\begin{bmatrix} x \\ y \end{bmatrix}$ equals the scalar product $T\begin{bmatrix} x \\ y \end{bmatrix}$. Find the smallest positive integral value of k.

6. Let $T =$ TAFTPQITR. If $2T$ is the length of the shortest leg of a right triangle whose three sides all have integer lengths, find the sum of all possible lengths of the hypotenuse of the triangle.

Relay 5

1. How many distinct permutations of the letters in the word MUALPHATHETA are there?

2. Let $T =$ TAFTPQITR. Find the smallest positive integer x such that $\frac{T}{100} + x$ is divisible by 1000.

3. Let $T =$ TAFTPQITR. If the sum $\sum_{n=1}^{T} ni^n$ can be written in the form $a + bi$, where i is the imaginary unit, find the sum of the digits of a.

4. Let $T =$ TAFTPQITR. Define a sequence by $a_1 = T$ and for $n \geq 2$, $a_n = 1 +$ the sum of the squares of the digits of a_{n-1}. What is the value of a_{2011}?

5. Let $T =$ TAFTPQITR. Find the distance from the point $(-7,5)$ to the line $3x - 4y = T$.

6. Let $T =$ TAFTPQITR. Find the area enclosed by a triangle with sides of length $T+1$, $T+2$, and $T+10$.