**#0 Alpha School Bowl**
**MAΘ National Convention 2011**

For the function $f\left(x\right)=\dfrac{x^2-9}{x^2+2x-15}$:

$A=$ the $y$-coordinate of the horizontal asymptote

$B=$ the $x$-coordinate of the vertical asymptote

$C=$ the $y$-coordinate of the removable discontinuity

$D=$ the $x$-coordinate of the removable discontinuity

Find the value of $\dfrac{ABD}{C}$.

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**#1 Alpha School Bowl**
**MAΘ National Convention 2011**

Consider the function $f(x)=\sum_{k=1}^{x}\log_7 k$ with domain of all positive integers.

Let $A_n =$ the largest integer value of $x$ such that $\lceil f(x)\rceil = n$, where $\lceil f(x)\rceil =$ the least integer $a$ satisfying $f(x)\le a$.

Find the value of $\sum_{n=1}^{6} A_n$ .

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**#2 Alpha School Bowl**
**MAΘ National Convention 2011**

The graph of $y = Ax^B$ passes through the points $(2,5)$ and $(4,25)$.

The graph of $y = C(D)^x$, $D > 0$, passes through the points $(2,5)$ and $(4,25)$.

The graph of $y = \dfrac{A(D)^{A+C}}{1+3(2)^{Bx}}$ passes through the points $(2,E)$ and $(4,F)$.

Find the value of $\dfrac{E-F}{EF}$.


**#2 Alpha School Bowl**
**MAΘ National Convention 2011**

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The graph of $y = \dfrac{A(D)^{A+C}}{1+3(2)^{Bx}}$ passes through the points $(2,E)$ and $(4,F)$.

Find the value of $\dfrac{E-F}{EF}$.

**#3 Alpha School Bowl**
**MAΘ National Convention 2011**

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -2 \\ 4 & 2 & 3 \end{bmatrix} \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} = \begin{bmatrix} -3 & 5 & -1 \\ -10 & 6 & -6 \\ 0 & 15 & 2 \end{bmatrix}$$

Find the value of $(B+C+D+E)^A + (FGH)^I$.

**#3 Alpha School Bowl**
**MAΘ National Convention 2011**

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Find the value of $(B+C+D+E)^A + (FGH)^I$.

## #4 Alpha School Bowl
## MAΘ National Convention 2011

$A = \left| -\frac{24}{7} + \frac{17}{5}i \right|$

A plane flying at a constant speed of 450 miles per hour is flying at a rate of $B$ feet per second.

$C =$ the number of distinct distributions of 10 identical widgets among 4 people

A sector of a circle with radius 5 and central angle of measure $187.2^\circ$ contains an area of $D\pi$.

Find the value of $\frac{AB}{CD}$.


## #4 Alpha School Bowl
## MAΘ National Convention 2011

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Find the value of $\frac{AB}{CD}$.

## #5 Alpha School Bowl
## MAΘ National Convention 2011

Two vectors $\vec{a}$ and $\vec{b}$ form an acute angle of $75^\circ$. Let $A = \dfrac{\left|\vec{a} \times \vec{b}\right|}{\vec{a} \cdot \vec{b}}$; $B =$ the length of the projection of $\vec{a}$ onto $\vec{b}$ if $\left|\vec{a}\right| = \dfrac{\sqrt{6} - \sqrt{2}}{4}$; and $C =$ the length of the projection of $\vec{a}$ onto $\vec{b}$ if $\left|\vec{a}\right| = \dfrac{\sqrt{6} + \sqrt{2}}{4}$. Find the smallest positive radian measure of angle $\theta$ such that $\sin\theta = \left(A \cdot B\right)^C$.


## #5 Alpha School Bowl
## MAΘ National Convention 2011

Two vectors $\vec{a}$ and $\vec{b}$ form an acute angle of $75^\circ$. Let $A = \dfrac{\left|\vec{a} \times \vec{b}\right|}{\vec{a} \cdot \vec{b}}$; $B =$ the length of the projection of $\vec{a}$ onto $\vec{b}$ if $\left|\vec{a}\right| = \dfrac{\sqrt{6} - \sqrt{2}}{4}$; and $C =$ the length of the projection of $\vec{a}$ onto $\vec{b}$ if $\left|\vec{a}\right| = \dfrac{\sqrt{6} + \sqrt{2}}{4}$. Find the smallest positive radian measure of angle $\theta$ such that $\sin\theta = \left(A \cdot B\right)^C$.

**#6 Alpha School Bowl**
**MAΘ National Convention 2011**

$A =$ the distance between the polar coordinates $\left(4, 57^\circ\right)$ and $\left(-3, 297^\circ\right)$

A population of bacteria grows at a rate proportional to its size. Initially the population consists of 100 bacteria. Three hours later the bacteria population is 2700. Let $B =$ the relative growth rate of the bacteria population.

For the matrix $\begin{bmatrix} 0 & 1 & -3 \\ 0 & 0 & 1 \\ 4 & -4 & -11 \end{bmatrix}$, let $C =$ the sum of its eigenvalues and $D =$ the product of its eigenvalues.

Find the value of $\left(A^2 + C + D\right)e^B$.

**#6 Alpha School Bowl**
**MAΘ National Convention 2011**

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**#7 Alpha School Bowl**
**MAΘ National Convention 2011**

$$A = \sum_{n=1}^{2011} 1$$

$$B = \sum_{n=1}^{2011} n$$

$$C = \sum_{n=1}^{2011} n^2$$

If $\sum_{n=1}^{\infty} \frac{n^2}{2^n} = \frac{C \cdot D}{A \cdot B}$, find the value of $D$.

**#7 Alpha School Bowl**
**MAΘ National Convention 2011**

$$A = \sum_{n=1}^{2011} 1$$

$$B = \sum_{n=1}^{2011} n$$

$$C = \sum_{n=1}^{2011} n^2$$

If $\sum_{n=1}^{\infty} \frac{n^2}{2^n} = \frac{C \cdot D}{A \cdot B}$, find the value of $D$.

## #8 Alpha School Bowl
## MAΘ National Convention 2011

Lucky Guess, poker-player extraordinaire, has just sat down at the casino at a table where the dealer is dealing 4-card stud poker. A full house is not possible in 4-card poker. However, other hands, such as flushes (only 4 cards) or straights (only 4 cards) are possible. The possible hands in 4-card poker, listed in descending order of their value in 5-card poker equivalent, are the following:

(A) Straight flush (four cards of the same suit of consecutive ranks)
(B) Four of a kind (four cards of the same rank)
(C) Flush (four cards of the same suit, not all of consecutive ranks)
(D) Straight (four cards of consecutive ranks, not all of the same suit)
(E) Three of a kind (three cards of the same rank, one card of another rank)
(F) Two pair (two cards of one rank, two cards of another rank)
(G) One pair (two cards of one rank, two cards of different other ranks)
(H) High card (none of the above)

List the above hands in descending order of value **in 4-card poker** by listing the corresponding letters. An ace may be counted as high in a straight (J, Q, K, A) or low in a straight (A, 2, 3, 4), but a straight may not have an ace as a middle card of the straight (e.g., K, A, 2, 3).


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## MAΘ National Convention 2011

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(A) Straight flush (four cards of the same suit of consecutive ranks)
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(C) Flush (four cards of the same suit, not all of consecutive ranks)
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(E) Three of a kind (three cards of the same rank, one card of another rank)
(F) Two pair (two cards of one rank, two cards of another rank)
(G) One pair (two cards of one rank, two cards of different other ranks)
(H) High card (none of the above)

List the above hands in descending order of value **in 4-card poker** by listing the corresponding letters. An ace may be counted as high in a straight (J, Q, K, A) or low in a straight (A, 2, 3, 4), but a straight may not have an ace as a middle card of the straight (e.g., K, A, 2, 3).

**#9 Alpha School Bowl**
**MAΘ National Convention 2011**

$A =$ the sum of the solutions to the equation $2\sqrt{x} = x - 15$

$B =$ the sum of the solutions to the equation $x - 5\sqrt{x} = -6$

$C =$ the sum of the solutions to the equation $3\sqrt{x} = x - 4$

$D =$ the sum of the solutions to the equation $2x + \sqrt{x} = 1$

Find the largest prime factor of the number $A \cdot B + C \cdot D$.

**#9 Alpha School Bowl**
**MAΘ National Convention 2011**

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$D =$ the sum of the solutions to the equation $2x + \sqrt{x} = 1$

Find the largest prime factor of the number $A \cdot B + C \cdot D$.

**#10 Alpha School Bowl**
**MAΘ National Convention 2011**

Solve for $X$:

$$\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{5}\right)+\tan^{-1}X=\tan^{-1}3$$

**#10 Alpha School Bowl**
**MAΘ National Convention 2011**

Solve for $X$:

$$\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{5}\right)+\tan^{-1}X=\tan^{-1}3$$

**#11 Alpha School Bowl**
**MAΘ National Convention 2011**

Find the domain, in interval notation, of the function:

$$f(x) = x(2-3x)^{-\frac{1}{2}} + \sqrt{3x+1} - \ln(2x-1) + \sin^{-1}(5x^2-1)$$

**#11 Alpha School Bowl**
**MAΘ National Convention 2011**

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$$f(x) = x(2-3x)^{-\frac{1}{2}} + \sqrt{3x+1} - \ln(2x-1) + \sin^{-1}(5x^2-1)$$

## #12 Alpha School Bowl
## MAΘ National Convention 2011

A rocket is launched upward at $v_0$ meters per second from a platform $h$ meters above ground. The equation for the rocket's height $s$ above ground at time $t$ seconds after launch is given by the equation $s(t) = -4.9t^2 + v_0 t + h$, where $s$ is in meters. Let $A =$ the number of seconds it would take the rocket to reach the ground if it is launched at 19.6 meters per second if the platform is 156.8 meters above ground, and let $B =$ the height, in meters, that the platform would need to be in order for the rocket to take 15 seconds to reach the ground if the rocket was launched at 29.4 meters per second. Find the value of $\frac{B}{A}$, written as a decimal.

## #12 Alpha School Bowl
## MAΘ National Convention 2011

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**#13 Alpha School Bowl**
**MAΘ National Convention 2011**

$A =$ the area enclosed by the ellipse with equation $5x^2 + 6y^2 - 50x + 12y + 101 = 0$

$B =$ the area enclosed by the parabola with equation $4x = y^2 + 6y + 13$ and the line with equation $x = 4$

Find the value of $A \cdot B$.

**#13 Alpha School Bowl**
**MAΘ National Convention 2011**

$A =$ the area enclosed by the ellipse with equation $5x^2 + 6y^2 - 50x + 12y + 101 = 0$

$B =$ the area enclosed by the parabola with equation $4x = y^2 + 6y + 13$ and the line with equation $x = 4$

Find the value of $A \cdot B$.

## #14 Alpha School Bowl
## MAΘ National Convention 2011

$$\frac{x^6+x^5+2x^4+x^3+2x^2+2x+3}{(x+1)(x^2+1)(x^4+1)}=\frac{A}{x+1}+\frac{B}{x^2+1}+\frac{C}{x^4+1}$$

Find the value of $A+2B+3C$.


## #14 Alpha School Bowl
## MAΘ National Convention 2011

$$\frac{x^6+x^5+2x^4+x^3+2x^2+2x+3}{(x+1)(x^2+1)(x^4+1)}=\frac{A}{x+1}+\frac{B}{x^2+1}+\frac{C}{x^4+1}$$

Find the value of $A+2B+3C$.