Answers:

1. B
2. C
3. C
4. C
5. A
6. E
7. D
8. A
9. D
10. A
11. C
12. A
13. A
14. D
15. B
16. D
17. D
18. E
19. C
20. B
21. D
22. E
23. C
24. C
25. B
26. A
27. C
28. C
29. C
30. A

Solutions:

1. $\int_{-3}^{3}\left(9-x^2\right)dx=2\int_{0}^{3}\left(9-x^2\right)dx=2\left(9x-\frac{1}{3}x^3\right)\Big|_0^3=2\left(9\cdot3-\frac{1}{3}\cdot3^3\right)=2\cdot18=36$

2. $\frac{1}{3}\int_{0}^{3}\left(x^3-5x\right)dx=\frac{1}{3}\left(\frac{1}{4}x^4-\frac{5}{2}x^2\right)\Big|_0^3=\frac{1}{3}\left(\frac{81}{4}-\frac{45}{2}\right)=\frac{1}{3}\left(-\frac{9}{4}\right)=-\frac{3}{4}$

3. Substituting $u=3x-1,\ du=3dx$ makes the integral $\frac{1}{9}\int(u+1)\sqrt{u}du$

$=\frac{1}{9}\int\left(u^{3/2}+u^{1/2}\right)du=\frac{1}{9}\left(\frac{2}{5}u^{5/2}+\frac{2}{3}u^{3/2}\right)+C=\frac{2}{9}u^{3/2}\left(\frac{1}{5}u+\frac{1}{3}\right)+C=\frac{2}{9}(3x-1)^{3/2}\left(\frac{9x+2}{15}\right)+C$

$=\frac{2(9x+2)(3x-1)^{3/2}}{135}+C$

4. Substituting $u=x+7,\ du=dx$ gives $\int_{-1}^{3}\left(f(x+7)+1\right)dx=\int_{6}^{10}\left(f(u)+1\right)du$

$=A+(10-6)=A+4$.

5. This is the area above the y-axis inside the unit circle with radius 10, so the value is $\frac{1}{2}\left(\pi(10)^2\right)=50\pi$.

6. $\int\tan^2 x dx=\int\left(\sec^2 x-1\right)dx=\tan x-x+C$

7. Since $x\geq2011$, by the Fund. Thm of Calc., $F'(x)=\frac{\left(x^2\right)^2\sin\left(x^2\right)}{1+\sqrt{x^2}}\cdot2x=\frac{2x^5\sin\left(x^2\right)}{1+x}$.

8. Since $v(t)>0$ when $2\leq t\leq4$, the total distance traveled is the displacement, which is $\int_{2}^{4}\left(2t^3+15\right)dt=\left(\frac{1}{2}t^4+15t\right)\Big|_2^4=(128+60)-(8+30)=150$.

9. $\pi\int_{0}^{1}\left((2+x)^2-\left(2+x^2\right)^2\right)dx=\pi\int_{0}^{1}\left(4x-3x^2-x^4\right)dx=\pi\int_{0}^{1}\left(\left(x^2+x+4\right)\left(x-x^2\right)\right)dx$

10. Graphically, we can see that $\int_{1}^{2}g(x)dx=2-\int_{0}^{1}\left(x^3+1\right)dx=2-\left(\frac{1}{4}x^4+x\right)\Big|_0^1$

$$=2-\left(\frac{1}{4}+1\right)=\frac{3}{4}$$

11. $\int_0^{\sqrt{3}}\frac{2x+3}{\sqrt{4-x^2}}dx=\int_0^{\sqrt{3}}\frac{2x}{\sqrt{4-x^2}}dx+\int_0^{\sqrt{3}}\frac{3}{\sqrt{4-x^2}}dx=\left(-2\sqrt{4-x^2}+3\sin^{-1}\frac{x}{2}\right)\Big|_0^{\sqrt{3}}$
$=(-2+\pi)-(-4)=\pi+2$

12. $\pi\int_{-1}^{0}\left(x\sqrt{x+1}\right)^2dx=\pi\int_{-1}^{0}\left(x^3+x^2\right)dx=\pi\left(\frac{1}{4}x^4+\frac{1}{3}x^3\right)\Big|_{-1}^{0}=-\pi\left(\frac{1}{4}-\frac{1}{3}\right)=\frac{\pi}{12}$

13. $\frac{1}{2}\left(\frac{1}{2^2}+\frac{2}{3^2}+\frac{1}{4^2}\right)=\frac{1}{2}\left(\frac{1}{4}+\frac{2}{9}+\frac{1}{16}\right)=\frac{77}{288}$

14. $10+\int_4^{11}3\sqrt{t+5}dt=10+\left(2(t+5)^{3/2}\right)\Big|_4^{11}=10+128-54=84$

15. Since $\int_2^{-2}f(t)dt=-6,\ \int_{-2}^{2}f(t)dt=6$, making 6 the last number in the first row. Similarly, the other number in the upper row would be -5. Now, since $\int_0^{-2}g(t)dt=4$ and g is odd, $\int_0^{2}g(t)dt=-\int_{-2}^{0}g(t)dt=\int_0^{-2}g(t)dt=4$, making 4 the last number in the second row. Similarly, the next to last number in the lower row would be 2. Finally, the middle number in the second row is 0 since that would be a definite integral with 0 as both the upper and lower limits. Thus, $6-5+4+2+0=7$.

16. $\int_1^2 2^{3x}dx=\left(\frac{2^{3x}}{3\ln 2}\right)\Big|_1^2=\frac{64-8}{3\ln 2}=\frac{56}{\ln 8}$

17. $2\pi\int_0^1(2-x)\left(2x-5x^{2/3}+3\right)dx=2\pi\int_0^1\left(-2x^2+5x^{5/3}+x-10x^{2/3}+6\right)dx$
$=2\pi\left(-\frac{2}{3}x^3+\frac{15}{8}x^{8/3}+\frac{1}{2}x^2-6x^{5/3}+6x\right)\Big|_0^1=2\pi\left(-\frac{2}{3}+\frac{15}{8}+\frac{1}{2}-6+6\right)=\frac{41\pi}{12}$

18. Letting $a=0,\ b=2,\ c=0.1,\ d=0.2,\ f(x)=(x-1)^2$, and $g(x)=x(2-x)$, we can see that A, B, C, and D are not necessarily true.

19. Since a and b are distinct, there are ${}_7P_2=42$ different choices of limits. Since f is

strictly increasing and odd, the only possibility of the integral equaling 0 is if the limits are negatives of each other, and there are 6 choices for those. Therefore, the sought probability is $\frac{42-6}{42}=\frac{36}{42}=\frac{6}{7}$.

20. $\int_0^5 (5-y)^2\,dy=\int_0^5\left(25-10y+y^2\right)dy=\left(25y-5y^2+\frac{1}{3}y^3\right)\Big|_0^5=125-125+\frac{125}{3}=\frac{125}{3}$

21. $\int_0^{\pi/2} x\cos x\,dx=\left(x\sin x+\cos x\right)\Big|_0^{\pi/2}=\left(\frac{\pi}{2}+0\right)-(0+1)=\frac{\pi}{2}-1$

22. $\frac{dy}{dx}=3x^2\left(y^2+4\right)\Rightarrow\int\frac{dy}{y^2+4}=\int 3x^2dx\Rightarrow\frac{1}{2}\tan^{-1}\frac{y}{2}=x^3+C$, which is not equivalent to any of the answer choices.

23. Since the graphs intersect when $x=0$ and $x=3$, $\bar{x}=\frac{\int_0^3 x\left(3x-x^2\right)dx}{\int_0^3\left(3x-x^2\right)dx}=\frac{\int_0^3\left(3x^2-x^3\right)dx}{\int_0^3\left(3x-x^2\right)dx}$

$$=\frac{\left(x^3-\frac{1}{4}x^4\right)\Big|_0^3}{\left(\frac{3}{2}x^2-\frac{1}{3}x^3\right)\Big|_0^3}=\frac{27-\frac{81}{4}}{\frac{27}{2}-9}=\frac{27/4}{9/2}=\frac{3}{2}.$$

24. Since the graph of f is concave down, the exact value will be larger than the Trapezoidal approximation, so $T<A$. Additionally, since Simpson's Rule gives exactly values for polynomials with degree less than 3, we must have $A=S$. Therefore, $T<S=A$.

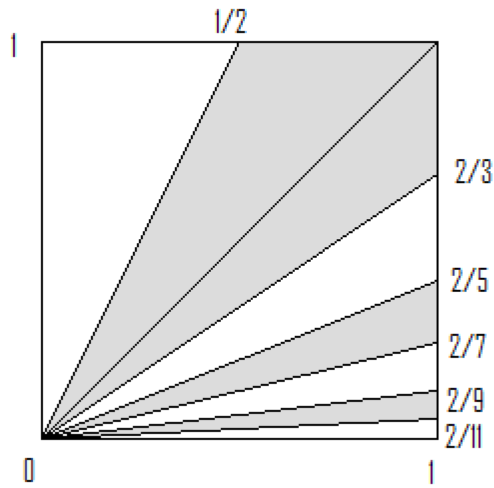
25. For this limit, $\Delta x=1$ and $x_0=0$. Therefore, $I=\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\left(\frac{i/n}{1+\left(i/n\right)}\right)=\int_0^1\frac{x}{1+x}dx$

$$=\int_0^1\left(1-\frac{1}{1+x}\right)dx=\left(x-\ln|1+x|\right)\Big|_0^1=1-\ln 2\text{, so } e^I=e^{1-\ln 2}=\frac{e}{2}.$$

26. The shaded regions constitute areas where x/y is closest to an odd integer (the regions moving clockwise continue to intersect the right side of the square at numbers of the form $2/\text{odd integer}$. Moving clockwise, the first triangle contains an

area of $\frac{1}{2}\cdot\frac{1}{2}\cdot 1=\frac{1}{4}$, and for the triangle to the right of the line $y=x$, each has an altitude of 1, so the total enclosed area is $\frac{1}{2}\cdot 1\cdot\left(\frac{1}{3}+2\left(\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\ldots\right)\right)$. Taking that $\tan^{-1}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\ldots$ for $-1\le x\le 1$, the innermost set of parentheses evaluates to $\tan^{-1}1-1+\frac{1}{3}=\frac{\pi}{4}-\frac{2}{3}$. Therefore, the sum of all shaded areas is

$$\frac{1}{4}+\frac{1}{2}\left(\frac{1}{3}+2\left(\frac{\pi}{4}-\frac{2}{3}\right)\right)=\frac{1}{4}+\frac{1}{2}\left(\frac{\pi}{2}-1\right)=\frac{\pi-1}{4}.$$



27. $\int_{-2}^{1}\left(f'(x)g(x)+f(x)g'(x)\right)dx=\int_{-2}^{1}\left(f(x)g(x)\right)'dx=f(x)g(x)\Big|_{-2}^{1}=1\cdot 3-(-5)\cdot 9=48$

28. $\int_{-1}^{2}\left(\frac{f'(x)g(x)-f(x)g'(x)}{\left(f(x)+g(x)\right)^2}\right)dx=\frac{f(x)}{f(x)+g(x)}\Bigg|_{-1}^{2}=\frac{7}{7+9}-\frac{1}{1+3}=\frac{7}{16}-\frac{1}{4}=\frac{3}{16}$

29. Interpreting the integral as area, $\int_{-2}^{1}|x+1|dx=\frac{1}{2}\cdot 1\cdot 1+\frac{1}{2}\cdot 2\cdot 2=\frac{5}{2}$.

30. $\int_{\pi/4}^{\pi/2}\frac{\cot^3 x}{\csc x}dx=\int_{\pi/4}^{\pi/2}\frac{\cos^3 x}{\sin^2 x}dx=\int_{\pi/4}^{\pi/2}\frac{\cos x\left(1-\sin^2 x\right)}{\sin^2 x}dx=\left(-\frac{1+\sin^2 x}{\sin x}\right)\Bigg|_{\pi/4}^{\pi/2}=\left(-\frac{1+1}{1}\right)$

$-\left(-\frac{1+1/2}{\sqrt{2}/2}\right)=-2+\sqrt{2}+\frac{\sqrt{2}}{2}=\frac{3\sqrt{2}-4}{2}$