

For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. What is the area bounded by the graphs of $y = 9 - x^2$ and the x -axis?

- A) 42 B) 36 C) 30 D) 24 E) NOTA

2. What is the average value of the function $f(x) = x^3 - 5x$ on the interval $[0, 3]$?

- A) 4 B) $\sqrt{3}$ C) $-\frac{3}{4}$ D) -2 E) NOTA

3. Evaluate: $\int x\sqrt{3x-1}dx$

- A) $\frac{9x-2}{2\sqrt{3x-1}} + C$ B) $\frac{x(3x-1)^{3/2}}{15} - \frac{2(3x-1)^{5/2}}{3} + C$ C) $\frac{2(9x+2)(3x-1)^{3/2}}{135} + C$
D) $\frac{x^2(3x-1)^{3/2}}{9} + C$ E) NOTA

4. If $\int_6^{10} f(x)dx = A$, express $\int_{-1}^3 (f(x+7)+1)dx$ in terms of A .

- A) $A+10$ B) $4A+32$ C) $A+4$ D) $2A+10$ E) NOTA

5. Evaluate: $\int_{-10}^{10} \sqrt{100-x^2}dx$

- A) 50π B) 25π C) 100π D) 200π E) NOTA

6. Evaluate: $\int \tan^2 x dx$

- A) $2\tan x \sec^2 x + C$ B) $x + \sec^2 x + C$ C) $\frac{\sin^3 x}{3\cos x} + C$ D) $\frac{1}{3}\tan^3 x + C$ E) NOTA

7. For $x \geq 2011$, $F(x) = \int_0^{x^2} \frac{t^2 \sin t}{1+\sqrt{t}} dt$. Find $F'(x)$.

- A) $\frac{4x \sin(x^2)}{(1+x)^2}$ B) $4x^3 \cos(x^2)$ C) $\frac{x^4 \sin(x^2)}{1+|x|}$ D) $\frac{2x^5 \sin(x^2)}{1+x}$ E) NOTA

8. The velocity of a particle moving along the x -axis is given by the function $v(t) = 2t^3 + 15$ for $t \geq 0$. Find the total distance traveled by the particle from $t = 2$ to $t = 4$.

- A) 150 B) 112 C) 72 D) 30 E) NOTA

9. Let R be the region bounded by the graphs of $y = x$ and $y = x^2$. Which of the following integrals is equal to the volume produced when R is rotated about the line $y = -2$?

- A) $\pi \int_0^1 (x - x^2 + 2)^2 dx$ B) $\pi \int_0^1 ((x-2)^2 - (x^2-2)^2) dx$ C) $\pi \int_0^1 (x^2 - x^4) dx$
D) $\pi \int_0^1 ((x^2 + x + 4)(x - x^2)) dx$ E) NOTA

10. Let $f(x) = x^3 + 1$ and let $g(x) = f^{-1}(x)$. Find the value of $\int_1^2 g(x) dx$.

- A) $\frac{3}{4}$ B) $\frac{4}{13}$ C) $\frac{5}{4}$ D) $\frac{4}{19}$ E) NOTA

11. Evaluate: $\int_0^{\sqrt{3}} \frac{2x+3}{\sqrt{4-x^2}} dx$

- A) 3π B) $2\pi + 1$ C) $\pi + 2$ D) 3 E) NOTA

12. The region in the third quadrant bounded by the graphs of $y = x\sqrt{x+1}$ and $y = 0$ is rotated about the x -axis. Find the volume of the resulting solid.

- A) $\frac{\pi}{12}$ B) $\frac{3\pi}{8}$ C) 2π D) $\frac{4\pi\sqrt{2}}{105}$ E) NOTA

13. Use the Trapezoidal Rule with two equal subdivisions to approximate $\int_1^3 \frac{1}{(x+1)^2} dx$.

- A) $\frac{77}{288}$ B) $\frac{619}{2400}$ C) $\frac{281}{1152}$ D) $\frac{109}{432}$ E) NOTA

14. Water is pumped into a tank at a rate of $3\sqrt{t+5}$ gallons/minute, with t representing the time, in minutes, elapsed after the pumping begins, with $0 \leq t \leq 20$. At time $t = 4$, the tank contains 10 gallons of water. How many gallons of water are in the tank at $t = 11$?

- A) 69 B) 74 C) 79 D) 84 E) NOTA

15. Let f and g be continuous functions that are even and odd, respectively. Find the sum of the missing entries in the following table that gives values of definite integrals:

x	-2	-1	0	1	2
$\int_{-x}^x f(t)dt$	-6		0	5	
$\int_0^x g(t)dt$	4	2			

- A) 5 B) 7 C) 9 D) 11 E) NOTA

16. Evaluate: $\int_1^2 2^{3x} dx$

- A) 180 B) $45\ln 8$ C) $\frac{100}{7}$ D) $\frac{56}{\ln 8}$ E) NOTA

17. Find the volume of the solid produced when the region bounded by the graphs of $y = 2x - 5x^{2/3} + 3$, where $0 \leq x \leq 1$, and the x -axis is rotated about the line $x = 2$.

- A) $\frac{7\pi}{12}$ B) $\frac{13\pi}{44}$ C) $\frac{57\pi}{44}$ D) $\frac{41\pi}{12}$ E) NOTA

18. If $\int_a^b f(x)dx \leq \int_a^b g(x)dx$, which of the following must be true?

- A) $f(x) \leq g(x)$ for all x in the interval $[a, b]$
 B) $f'(x) \leq g'(x)$ for all x in the interval (a, b)
 C) $f''(x) \leq g''(x)$ for all x in the interval (a, b)
 D) $\int_c^d f(x)dx \leq \int_c^d g(x)dx$ where $a < c < d < b$ E) NOTA

19. Let f be a continuous, strictly increasing, odd function, where $f(x) > 0$ whenever $x > 0$, and let $S = \{-3, -2, -1, 0, 1, 2, 3\}$. Two distinct elements a and b are selected from S . Find the probability that $\int_a^b f(x)dx \neq 0$.

- A) $\frac{4}{7}$ B) $\frac{5}{7}$ C) $\frac{6}{7}$ D) 1 E) NOTA

20. Let S be a solid whose base is the region bounded by the graphs of $y=5-x$, $y=0$, and $x=0$. If the cross-sections of this solid perpendicular to the y -axis are squares, find the volume of S .

- A) $\frac{625}{4}$ B) $\frac{125}{3}$ C) $\frac{625\pi}{4}$ D) $\frac{125\pi}{3}$ E) NOTA

21. Evaluate: $\int_0^{\pi/2} x \cos x dx$

- A) 1 B) $1 - \frac{\sqrt{2}}{2}$ C) 2 D) $\frac{\pi}{2} - 1$ E) NOTA

22. Which of the following is a solution to the differential equation $\frac{dy}{dx} = 3(xy)^2 + 12x^2$?

- A) $y = 2\sin(x^3 + 1)$ B) $y = 5 - \frac{x}{3(x^2 + 4)^2}$ C) $y = 5x^3 - 2011$ D) $y = \sqrt{10e^{x^3} - 4}$

E) NOTA

23. Find the x -coordinate of the centroid of the region bounded by the graphs of $y = x + 2$ and $y = 4x - x^2 + 2$. Assume the region has uniform density.

- A) $\frac{3}{10}$ B) $\frac{3}{4}$ C) $\frac{3}{2}$ D) $\frac{9}{4}$ E) NOTA

24. Let R be a region bounded by a nonnegative quadratic function f and the x -axis on the interval $[a, b]$ with $a < b$. Define A to be the exact area of R , T to be the Trapezoidal Rule estimate of A with 2010 equal partitions of $[a, b]$, and S to be the Simpson's Rule estimate of A with 2008 equal partitions of $[a, b]$. If $f'(x) > 0$ and $f''(x) < 0$ on the interval (a, b) , which of the following must be true?

- A) $T < A < S$ B) $S \leq A < T$ C) $T < S = A$ D) $A \leq S < T$ E) NOTA

25. Let $I = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \left(\frac{i/n}{1 + (i/n)} \right)$. Find the value of e^I .

- A) e B) $\frac{e}{2}$ C) $\frac{e}{3}$ D) $\frac{e}{4}$ E) NOTA

26. Two real numbers x and y are randomly chosen from the interval $(0,1)$. What is the probability that the closest integer to $\frac{x}{y}$ is odd?

- A) $\frac{\pi-1}{4}$ B) $\frac{1}{2}$ C) $\frac{\pi-2}{2}$ D) $\frac{6}{\pi^2}$ E) NOTA

Use the following table for questions 27-28. Assume f and g are twice-differentiable.

x	-2	-1	0	1	2
$f(x)$	-5	1	1	1	7
$g(x)$	9	3	1	3	9
$f'(x)$	11	2	-1	2	11
$g'(x)$	-8	-4	0	4	8

27. Evaluate: $\int_{-2}^1 (f'(x)g(x) + f(x)g'(x)) dx$

- A) 16 B) 32 C) 48 D) 64 E) NOTA

28. Evaluate: $\int_{-1}^2 \left(\frac{f'(x)g(x) - f(x)g'(x)}{(f(x) + g(x))^2} \right) dx$

- A) $\frac{1}{16}$ B) $\frac{1}{8}$ C) $\frac{3}{16}$ D) $\frac{1}{4}$ E) NOTA

29. Evaluate: $\int_{-2}^1 |x+1| dx$

- A) $\frac{1}{2}$ B) $\frac{3}{2}$ C) $\frac{5}{2}$ D) $\frac{7}{2}$ E) NOTA

30. Evaluate: $\int_{\pi/4}^{\pi/2} \frac{\cot^3 x}{\csc x} dx$

- A) $\frac{3\sqrt{2}-4}{2}$ B) $\frac{3\sqrt{2}-4}{4}$ C) $\frac{3\sqrt{2}-2}{2}$ D) $\frac{3\sqrt{2}-2}{4}$ E) NOTA