Answers:

1. A
2. E
3. D
4. C
5. B
6. B
7. A
8. B
9. B
10. C
11. A
12. E
13. B
14. E
15. A
16. D
17. B
18. D
19. A
20. E
21. C
22. D
23. C
24. E
25. D
26. B
27. C
28. B
29. C
30. B

Solutions:

1. $m=\frac{3(2+27+65)-(1+3+5)(2+9+13)}{3(1+9+25)-(1+3+5)^2}=\frac{66}{24}=\frac{11}{4}$ and $b=8-\frac{11}{4}(3)=-\frac{1}{4}$, so $b-m=-\frac{1}{4}-\frac{11}{4}=-3$

2. $x^4+8x^3+7x^2-72x-144=(x+3)(x-3)(x+4)^2$ which is positive when x is in $(-\infty,-4)\cup(-4,-3)\cup(3,\infty)$

3. $(17 \text{ mod } 4)-(20511 \text{ mod } 3)((449 \text{ mod } 7)-(9901 \text{ mod } 5))=1-0(1-1)=1$

4. For $\lambda(16)$, we must examine powers of odd integers less than 16. Also, $1 \text{ mod } n=1$ for all integers $n\geq 2$. The smallest integer exponents are $3^4 \text{ mod } 16=1$, $5^4 \text{ mod } 16 =25^2 \text{ mod } 16=9^2 \text{ mod } 16=1$, $7^2 \text{ mod } 16=1$, $9^2 \text{ mod } 16=1$, $11^4 \text{ mod } 16 =121^2 \text{ mod } 16=9^2 \text{ mod } 16=1$, $13^4 \text{ mod } 16=169^2 \text{ mod } 16=9^2 \text{ mod } 16=1$, and $15^2 \text{ mod } 16=1$. Since the smallest powers for these integers are all 2 or 4, and that $x \text{ mod } 16=1 \Rightarrow x^n \text{ mod } 16=1$, we must have $\lambda(16)=4$.

5. $F\left(\frac{25}{4},\frac{16}{3}\right)=F\left(\frac{21}{4},\frac{13}{3}\right)+2=F\left(\frac{17}{4},\frac{10}{3}\right)+4=F\left(\frac{13}{4},\frac{7}{3}\right)+6=F\left(\frac{9}{4},\frac{4}{3}\right)+8=F\left(\frac{5}{4},\frac{1}{3}\right)$ $+10=F\left(\frac{1}{4},\frac{1}{3}\right)+11=\frac{1}{4}-\frac{1}{3}+11=\frac{3}{12}-\frac{4}{12}+\frac{132}{12}=\frac{131}{12}$

6. By Descartes' Rule of Signs, w has either 4 positive roots, 2 positive and 2 imaginary roots, or 4 imaginary roots. I is clearly false. II is false if the roots fall as in the first categorization of roots. III is true as in the third categorization of roots.

7. Plugging in $x=2$ gives $f(2)+2f(-1)=2$. Plugging in $x=-1$ gives $f(-1)+2f\left(\frac{1}{2}\right)=-1$. Plugging in $x=\frac{1}{2}$ gives $f\left(\frac{1}{2}\right)+2f(2)=\frac{1}{2}$. Adding these 3 equations together and dividing by 3 gives $f\left(\frac{1}{2}\right)+f(2)+f(-1)=\frac{1}{2}$, which when combined with the third equation makes $f(2)=f(-1)$, which when combined with the first equation makes $f(2)=\frac{2}{3}$.

8. $h(n)=\log_2 n$, so $h(2^k)=\log_2 2^k=k$, meaning $\sum_{k=2}^{10} h(2^k)=\sum_{k=2}^{10} k=\frac{10\cdot 11}{2}-1=54$

9. The range of the inverse is the domain of the function, which is $(-\infty,3)\cup(3,\infty)$.

10. $Q(0)=Q(0+0)=Q(0)^2 \Rightarrow Q(0)=0$ or $Q(0)=1$. However, if $Q(0)=0$, then for any x, $Q(x)=Q(x+0)=Q(x)Q(0)=0$, which contradicts the given assumption. Therefore I is false. There is no reason III can't be true. Finally, $Q(-2)\cdot Q(2)=Q(-2+2)=Q(0)$ $=1$, so II is true.

11. $0=(x+3)^3+2(x+3)^2-8(x+3)=(x+3)\left((x+3)^2+2(x+3)-8\right)$
$=(x+3)(x+3+4)(x+3-2)$, so the sum of the solutions is $-3-7-1=-11$

12. $M(x)=(x^2+1)(x^2-2x+5)(x+3)=x^5+x^4+16x^2-x+15$, so $(b-a)(c-e)$
$=(0-1)(16-15)=-1$

13. $|17-2x|>4 \Rightarrow 17-2x>4$ or $17-2x<-4 \Rightarrow 2x<13$ or $2x>21 \Rightarrow x<6.5$ or $x>10.5$, so $a+b=6.5+10.5=17$

14. $0=4x^2-25y^2-24x-50y+11=4(x-3)^2-25(y+1)^2$, so the graph consists of two lines intersecting at the point $(3,-1)$.

15. For this parabola, $\frac{1}{4p}=|a|=\frac{1}{3}\Rightarrow 4p=3$, and since the latus lectum has length $4p$, the sought length is 3.

16. $C(P(4))=C\left(\binom{4}{2}\right)=C(6)=5!$

17. $P(C(4))=P(3!)=P(6)=\binom{6}{2}=15$

18. $y=\ln(g(x))=\ln(ae^{bx})=\ln a+bx$, which is a linear relationship

19. $C = 1.8C + 32 \Rightarrow 0.8C = -32 \Rightarrow C = -40$, so the sum of the digits of the absolute value is $4 + 0 = 4$

20. I is neither even nor odd since the function is only valid for positive x-values. II is even since plugging in a positive or negative value with the same magnitudes gives the same value. III is neither even nor odd because only the cubed term would change signs when plugging in a negative value.

21. $K = C + 273 = \frac{5}{9}(F - 32) + 273$, which is generated by an upward shift of 273 units

22. $S(-1) + S(2) + S(7) = 1 + 2^2 + 2 \cdot 2 + 3 + 7^3 - 7 = 1 + 4 + 4 + 3 + 343 - 7 = 348$

23. $0 > \frac{x+1}{x-3} - 2 = \frac{x + 1 - 2(x-3)}{x-3} = \frac{-x+7}{x-3}$, which holds for $x < 3$ or $x > 7$. Therefore, $b - a = 7 - 3 = 4$.

24. $u(v(x)) = u(c\ln(dx)) = ae^{b(c\ln(dx))} = a(dx)^{bc} = ad^{bc}x^{bc}$ and $v(u(x)) = v(ae^{bx})$ $= c\ln(dae^{bx}) = c\ln(da) + cbx$. To make the two compositions equal, because the second composition is linear, we must have $bc = 1$ to make the first linear. This makes the first composition adx and the second $x + c\ln(da)$. To make the coefficients of x equal, $da = 1$ must also be true. This would make both compositions equal to x. So to have the two compositions equal, we must have $bc = 1 = ad$. The answer must be if and only if both conditions from B and C; D is not enough since that would include, for example, if bc and ad both equaled 2.

25. Since $f(2x) = \log_2 x$, $f(x) = \log_2 \frac{x}{2} = \log_2 x - \log_2 2 = -1 + \log_2 x$.

26. Since the three roots form an arithmetic progression and their sum is 12, 4 must be a root. Therefore, $0 = H(4) = 4^3 - 12(4)^2 + 37(4) + G = 64 - 192 + 148 + G = 20 + G$, so $G = -20$.

27. Using long division, $\frac{x^2 - x - 2}{x+2} = x - 3 + \frac{4}{x+2}$, so the oblique asymptote is $y = x - 3$.

28. In order for the two solutions to be distinct and real , we must have $0 < (a-3)^2 - 4 \cdot 1 \cdot a = a^2 - 10a + 9 = (a-1)(a-9)$, so $a < 1$ or $a > 9$. For the roots to be

positive, we must have that their sum and product are positive, meaning that $a-3<0$ and $a>0$, so $0<a<3$. The intersection of these two inequalities is $0<a<1$.

29. $f(x)=1+\frac{1}{x}$, so $1-\frac{1}{x}=f\left(g\left(1-\frac{1}{x}\right)\right)=1+\frac{1}{g\left(1-\frac{1}{x}\right)}\Rightarrow -\frac{1}{x}=\frac{1}{g\left(1-\frac{1}{x}\right)}$

$\Rightarrow g\left(1-\frac{1}{x}\right)=-x$

30. $0=\log_b\left(\frac{1+\sqrt{1-x^2}}{x}\right)\Rightarrow 1=b^0=\frac{1+\sqrt{1-x^2}}{x}\Rightarrow x-1=\sqrt{1-x^2}\Rightarrow x^2-2x+1=1-x^2$

$\Rightarrow 0=2x^2-2x=2x(x-1)$, so $x=1$ or $x=0$, but the second number is not in the domain, so we must have $y^{-1}(0)=1$.