Answers:

1. B
2. D
3. B
4. A
5. D
6. B
7. D
8. A
9. B
10. B
11. C
12. C
13. E
14. C
15. D
16. A
17. B
18. B
19. A
20. B
21. B
22. B
23. B
24. C
25. C
26. D
27. A
28. A
29. C
30. E

Solutions:

1. $\begin{bmatrix}6 & 7\\2 & 1\end{bmatrix}-\begin{bmatrix}5 & -2\\3 & 4\end{bmatrix}\begin{bmatrix}1 & 4\\2 & -1\end{bmatrix}=\begin{bmatrix}6 & 7\\2 & 1\end{bmatrix}-\begin{bmatrix}5-4 & 20+2\\3+8 & 12-4\end{bmatrix}=\begin{bmatrix}6 & 7\\2 & 1\end{bmatrix}-\begin{bmatrix}1 & 22\\11 & 8\end{bmatrix}=\begin{bmatrix}5 & -15\\-9 & -7\end{bmatrix}$

2. $\begin{bmatrix}a & b & c\\d & e & f\\g & h & i\end{bmatrix}=\begin{bmatrix}4 & 11 & 6\\7 & 4 & 2\\8 & -3 & 9\end{bmatrix}\cdot\begin{bmatrix}3 & 1 & 1\\-2 & 7 & 8\\4 & 6 & 5\end{bmatrix}=\begin{bmatrix}12-22+24 & 4+77+36 & 4+88+30\\21-8+8 & 7+28+12 & 7+32+10\\24+6+36 & 8-21+54 & 8-24+45\end{bmatrix}$

$=\begin{bmatrix}14 & 117 & 122\\21 & 47 & 49\\66 & 41 & 29\end{bmatrix}$, so $b-a-e-i=117-14-47-29=27$

3. The area would be the magnitude of their cross product. The cross product is $\vec{u}\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\3 & 2 & 2\\1 & -4 & 5\end{vmatrix}=18\vec{i}-13\vec{j}-14\vec{k}$, so the magnitude is $\sqrt{18^2+(-13)^2+(-14)^2}$ $=\sqrt{324+169+196}=\sqrt{689}$.

4. The vectors are perpendicular is their dot product is 0. $0=\vec{u}\cdot\vec{v}=\left(e^{3x/2}\right)^2-\left(e^x\right)^2$ $-2\left(e^{x/2}\right)^2+2=e^{3x}-e^{2x}-2e^x+2=\left(e^x-1\right)\left(e^x-\sqrt{2}\right)\left(e^x+\sqrt{2}\right)\Rightarrow x=0$ or $x=\ln\sqrt{2}$, so the sum of the solutions is $\ln\sqrt{2}$.

5. Both transposing and finding the inverse of products reverse the order of the corresponding matrices. Therefore, $\left((AB)^T\right)^{-1}=\left(B^TA^T\right)^{-1}=\left(A^T\right)^{-1}\left(B^T\right)^{-1}$.

6. The magnitude of the vector is $\sqrt{8^2+(-6)^2}=\sqrt{64+36}=\sqrt{100}=10$, so the unit vector (has magnitude 1) is $\left\langle\frac{8}{10},\frac{-6}{10}\right\rangle=\left\langle\frac{4}{5},-\frac{3}{5}\right\rangle$.

7. The coefficient determinant is $\begin{vmatrix} 2 & 5 & -3 \\ 5 & -1 & 2 \\ 1 & 2 & -1 \end{vmatrix} = 2+10-30-3-8+25 = -4$, so by Cramer's Rule, the solution is $x = \dfrac{\begin{vmatrix} 8 & 5 & -3 \\ 3 & -1 & 2 \\ 6 & 2 & -1 \end{vmatrix}}{-4} = \dfrac{8+60-18-18-32+15}{-4} = -\dfrac{15}{4}$, $y = \dfrac{\begin{vmatrix} 2 & 8 & -3 \\ 5 & 3 & 2 \\ 1 & 6 & -1 \end{vmatrix}}{-4} = \dfrac{-6+16-90+9-24+40}{-4} = \dfrac{55}{4}$, and $z = \dfrac{\begin{vmatrix} 2 & 5 & 8 \\ 5 & -1 & 3 \\ 1 & 2 & 6 \end{vmatrix}}{-4}$ $= \dfrac{-12+15+80+8-12-150}{-4} = \dfrac{71}{4}$. So the solution is $\left(-\dfrac{15}{4}, \dfrac{55}{4}, \dfrac{71}{4}\right)$.

8. The determinant must be 0 for the matrix to be singular. Since in the matrix for choice A the second column multiplied by 2 equals the third column, that matrix has a determinant of 0.

9. $A\begin{bmatrix} -2 & 3 \\ 5 & 1 \end{bmatrix} = 17\begin{bmatrix} 4 & -1 \\ 6 & 5 \end{bmatrix} = \begin{bmatrix} 68 & -17 \\ 102 & 85 \end{bmatrix} \Rightarrow A = \dfrac{1}{-17}\begin{bmatrix} 68 & -17 \\ 102 & 85 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -5 & -2 \end{bmatrix}$ $= \begin{bmatrix} -4 & 1 \\ -6 & -5 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -5 & -2 \end{bmatrix} = \begin{bmatrix} -4-5 & 12-2 \\ -6+25 & 18+10 \end{bmatrix} = \begin{bmatrix} -9 & 10 \\ 19 & 28 \end{bmatrix}$

10. θ satisfies $\cos\theta = \dfrac{\langle 3,1,-4\rangle \cdot \langle 2,-1,5\rangle}{\sqrt{3^2+1^2+(-4)^2}\sqrt{2^2+(-1)^2+5^2}} = \dfrac{6-1-20}{\sqrt{9+1+16}\sqrt{4+1+25}} = -\dfrac{15}{\sqrt{780}}$, so $\tan^2\theta - 1 = \sec^2\theta - 2 = \left(-\dfrac{\sqrt{780}}{15}\right)^2 - 2 = \dfrac{780-450}{225} = \dfrac{330}{225} = \dfrac{22}{15}$.

11. A plane would be perpendicular to the given plane if their normal vectors were also perpendicular, meaning their dot product is 0. The normal vector's components are the coefficients of the variables, so if A, B, and C are the coefficients of x, y, and z, respectively, in the sought plane, then $0 = \langle 4,-3,2\rangle \cdot \langle A,B,C\rangle = 4A-3B+2C$. The only plane of those given that satisfies this equation is choice C.

12. The trace is the sum of the entries on the main diagonal, so the trace is $\ln 5 - \ln 3$ $+e^2 = \ln 5/3 + \ln e^{e^2} = \ln 5e^{e^2}/3$.

13. $\begin{vmatrix} 2 & -3 & 0 & -2 \\ 3 & -1 & 1 & 1 \\ -2 & 1 & -1 & 0 \\ 0 & 3 & 1 & 2 \end{vmatrix} = 2\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 0 \\ 3 & 1 & 2 \end{vmatrix} + 3\begin{vmatrix} 3 & 1 & 1 \\ -2 & -1 & 0 \\ 0 & 1 & 2 \end{vmatrix} + 2\begin{vmatrix} 3 & -1 & 1 \\ -2 & 1 & -1 \\ 0 & 3 & 1 \end{vmatrix} = 2(2+0+1+3-0$
$-2)+3(-6+0-2-0-0+4)+2(3+0-6-0+9-2)=8-12+8=4$

14. This matrix satisfies $A=-A^T$, which is the definition of skew-symmetric. It isn't invertible since its determinant is 0; it isn't symmetric since $a_{12} \neq a_{21}$. Radial doesn't apply.

15. $0=(x^2-4)(x+3)-2(x+2)=(x+2)(x^2+x-8)$, and since the discriminant of the quadratic is positive, all three solutions are real. The product of the two in the quadratic is -8, so the product of all three is $(-8)(-2)=16$.

16. The magnitudes of the two vectors, which represent the length of two of the edges of the prism, are 8 and 4, and since the volume of the prism is 64, the other length must be 2. Therefore, the longest diagonal has length $\sqrt{2^2+4^2+8^2}=\sqrt{4+16+64}$ $=\sqrt{84}=2\sqrt{21}$.

17. $\text{proj}_{\vec{b}}\vec{a}=\frac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|^2}\vec{b}=\frac{-18-10}{(-6)^2+2^2}\langle -6,2\rangle=-\frac{7}{10}\langle -6,2\rangle=\left\langle \frac{21}{5},-\frac{7}{5}\right\rangle$

18. $\sqrt{\sin^2 23^\circ+\sin^2 42^\circ+\sin^2 48^\circ+\sin^2 67^\circ}=\sqrt{\sin^2 23^\circ+\sin^2 42^\circ+\cos^2 42^\circ+\cos^2 23^\circ}$ $=\sqrt{1+1}=\sqrt{2}$

19. $\vec{a}\times\vec{b}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & -3 \\ 0 & 2 & 0 \end{vmatrix}=6\vec{i}$, which points in the positive x-axis direction

20. $\begin{vmatrix} \cos 2x & -1 \\ i\sin 2x & 1 \end{vmatrix}=\cos 2x+i\sin 2x=\text{c}is\,2x=e^{2ix}$

21. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}^{2011}=2^{2011}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=2^{2011}I$

22. $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)=\begin{vmatrix}2 & -1 & 7\\ 0 & 2 & -1\\ 4 & -3 & 5\end{vmatrix}=20+4+0-56-6-0=-38$

23. $\begin{bmatrix}10.8 & 4.1\\ 62.4 & 39.3\end{bmatrix}+\begin{bmatrix}1072.3 & 2.4\\ 21.8 & 96.1\end{bmatrix}=\begin{bmatrix}1083.1 & 6.5\\ 84.2 & 135.4\end{bmatrix}$

24. Transposing a matrix doesn't change its determinant, so the answer is 36.

25. $A^{-1}=\frac{1}{-15-14}\begin{bmatrix}5 & -2\\ -7 & -3\end{bmatrix}=-\frac{1}{29}\begin{bmatrix}5 & -2\\ -7 & -3\end{bmatrix}$ and $B^{-1}=\frac{1}{18+8}\begin{bmatrix}3 & -4\\ 2 & 6\end{bmatrix}=\frac{1}{26}\begin{bmatrix}3 & -4\\ 2 & 6\end{bmatrix}$, so

$$A^{-1}B^{-1}=\left(-\frac{1}{29}\right)\left(\frac{1}{26}\right)\begin{bmatrix}5 & -2\\ -7 & -3\end{bmatrix}\begin{bmatrix}3 & -4\\ 2 & 6\end{bmatrix}=-\frac{1}{754}\begin{bmatrix}15-4 & -20-12\\ -21-6 & 28-18\end{bmatrix}=-\frac{1}{754}\begin{bmatrix}11 & -32\\ -27 & 10\end{bmatrix}$$

26. The rotation matrix for counterclockwise rotation through angle θ is

$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, so for 60°, the matrix is $\begin{bmatrix}1/2 & -\sqrt{3}/2\\ \sqrt{3}/2 & 1/2\end{bmatrix}$.

27. $\sqrt{6^2+(-4)^2+3^2}=\sqrt{36+16+9}=\sqrt{61}$

28. $|AB|=|A||B|=(x+6)(8+x)$, which when considered as a parabola opening upward, has zeros of -8 and -6. Therefore, the vertex occurs when $x=-7$, and the minimum value is $(-7+6)(8-7)=-1$.

29. AB is not defined because the number of columns of A is 2 and the number of rows of B is 3. CB and BC are both defined because ones' outer and the other's inner dimension are the same in both respects. DB is defined because the number of columns of D and the number of rows of B are both 3. DC is not defined because the number of columns of D is 3 and the number of rows of C is 2. Therefore, three of the products are defined.

30. $A^{-1}=\frac{1}{-9-4x}\begin{bmatrix}-3 & -4\\ -x & 3\end{bmatrix}$, which will only equal A if the scalar is -1. So

$-9-4x=-1\Rightarrow -4x=8\Rightarrow x=-2$.