Answers:

1. C
2. C
3. D
4. C
5. D
6. A
7. C
8. B
9. D
10. E
11. D
12. B
13. C
14. A
15. B
16. A
17. D
18. A
19. C
20. A
21. B
22. C
23. D
24. D
25. D
26. B
27. B
28. A
29. C
30. C

Solutions:

1. Any angle coterminal with $-\pi/3$ and an r-value of -4 would be equivalent, so III is equivalent. An angle π away from one of these angles with an r-value of 4 would also work, so II is equivalent. These are the only ways to get an equivalent point, so I, IV, and V are not equivalent to the original point.

2. $r=\sqrt{(-2)^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$, so $\cos\theta=\frac{-2}{2\sqrt{2}}=-\frac{1}{\sqrt{2}}$ and $\sin\theta=\frac{-2}{2\sqrt{2}}$ $-\frac{1}{\sqrt{2}}\Rightarrow\theta=\frac{5\pi}{4}$. Therefore, $\left(2\sqrt{2},5\pi/4\right)$ or any other coterminal angle is one way to write the point, making C an acceptable way to write the point (and making D an unacceptable way to write the point). By the discussion in the last problem, $\left(-2\sqrt{2},\pi/4\right)$ or any other coterminal angle is another way to write the point, so A and B are unacceptable ways to write the point.

3. $\left(\left(2cis47^\circ\right)\left(3cis18^\circ\right)\right)^3=\left(6cis65^\circ\right)^3=6^3cis\left(3\cdot 65^\circ\right)=216cis195^\circ$, so the angle argument is 195°.

4. $0=x^4+2x^2y^2+y^4-4x^2+4y^2=\left(x^2+y^2\right)^2-4x^2+4y^2=r^4-4r^2\cos^2\theta+4r^2\sin^2\theta$ $\Rightarrow r^4=4r^2\left(\cos^2\theta-\sin^2\theta\right)=4r^2\cos2\theta\Rightarrow r^2=4\cos2\theta$, which is a lemniscate

5. $z=\frac{a}{2}+be^{i\theta}+\frac{a}{2}e^{2i\theta}$ is the form in the complex plane for $r=b+a\cos\theta$, so $a=2$ and $b=3$, so the base area is $\left(3^2+0.5\cdot 2^2\right)\pi=11\pi$, so $\frac{11\pi}{\pi/2}=22$ snails could cover the floor of the cage.

6. $r=\frac{4}{2+\cos\theta}=\frac{2}{1+\frac{1}{2}\cos\theta}$, so the eccentricity of the conic is $\frac{1}{2}$, making it an ellipse

7. Written in this form, the numerator, 2, is half the length of the latus rectum, meaning the latus rectum length is 4.

8. See the work for question 6—the eccentricity is $1/2$.

9. $r=\theta$ is also known as the Spiral of Archimedes.

10. Using the Law of Cosines, $c^2=4^2+\sqrt{2}^2-2\cdot 4\cdot\sqrt{2}\cos\left(\frac{9\pi}{4}-\frac{5\pi}{3}\right)=16+2-8\sqrt{2}\cos\frac{7\pi}{12}$

$18-8\sqrt{2}\cdot\frac{\sqrt{6}-\sqrt{2}}{4}=18-4\sqrt{3}+4=22-4\sqrt{3}\Rightarrow c=\sqrt{22-4\sqrt{3}}$, which is not equivalent to any of the choices.

11. $r=(\cos 4\theta)(\sin 4\theta)=\frac{1}{2}(2\cos 4\theta\sin 4\theta)=\frac{1}{2}\sin 8\theta$, which has $2\cdot 8=16$ petals since 8 is even

12. $\vec{u_c}=\left\langle -3\cos\frac{\pi}{3},-3\sin\frac{\pi}{3}\right\rangle=\left\langle -\frac{3}{2},-\frac{3\sqrt{3}}{2}\right\rangle$ and $\vec{v_c}=\left\langle -6\cos\frac{3\pi}{4},-6\sin\frac{3\pi}{4}\right\rangle=\left\langle 3\sqrt{2},-3\sqrt{2}\right\rangle$,

so $\vec{u_c}\cdot\vec{v_c}=\left(-\frac{3}{2}\right)\left(3\sqrt{2}\right)+\left(-\frac{3\sqrt{3}}{2}\right)\left(-3\sqrt{2}\right)=-\frac{9\sqrt{2}}{2}+\frac{9\sqrt{6}}{2}=\frac{9\left(\sqrt{6}-\sqrt{2}\right)}{2}$

13. The points are in order of being connected, and their rectangular coordinates are $\left(\sqrt{3},1\right)$, $\left(-3/2, 3\sqrt{3}/2\right)$, $\left(-2\sqrt{2},-2\sqrt{2}\right)$, and $\left(5\sqrt{3}/2, -5/2\right)$, respectively. Using the Shoelace method, the enclosed area is

$$\begin{array}{r|cc|l} & \sqrt{3} & 1 & \\ -3/2 & -3/2 & 3\sqrt{3}/2 & 9/2 \\ -3\sqrt{6} & -2\sqrt{2} & -2\sqrt{2} & 3\sqrt{2} \\ -5\sqrt{6} & 5\sqrt{3}/2 & -5/2 & 5\sqrt{2} \\ -5\sqrt{3}/2 & \sqrt{3} & 1 & 5\sqrt{3}/2 \\ -\frac{3}{2}-8\sqrt{6}-\frac{5\sqrt{3}}{2} & & & \frac{9}{2}+8\sqrt{2}+\frac{5\sqrt{3}}{2} \end{array}$$

$\Rightarrow A=\frac{1}{2}\left|-\frac{3}{2}-8\sqrt{6}-\frac{5\sqrt{3}}{2}-\frac{9}{2}-8\sqrt{2}-\frac{5\sqrt{3}}{2}\right|=\frac{8\sqrt{6}+8\sqrt{2}+5\sqrt{3}+6}{2}$

14. The values of r go to 0 as θ approaches $\frac{3\pi}{2}$, so this is from underneath, meaning the cust points upward.

15. $i^{-i}=\left(e^{i\left(\frac{\pi}{2}+2k\pi\right)}\right)^{-i}=e^{\frac{\pi}{2}+2k\pi}$, and the principal value of this is $e^{\frac{\pi}{2}+2\cdot 0\pi}=e^{\frac{\pi}{2}}$

16. Since $4>2$, the graph of $r=2+4\cos\theta$ will have an inner loop.

17. These two circle overlap in a region whose enclosed area is $2\left(\frac{1}{4}\pi\cdot\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^3\right)$ $=2\left(\frac{\pi}{16}-\frac{1}{8}\right)=\frac{\pi-2}{8}$. The total enclosed area is $2\left(\pi\left(\frac{1}{2}\right)^2\right)-\frac{\pi-2}{8}=\frac{3\pi+2}{8}$, so the probability is $\frac{\frac{\pi-2}{8}}{\frac{3\pi+2}{8}}=\frac{\pi-2}{3\pi+2}$.

18. Let $\theta=\tan^{-1}\left(-\frac{7}{24}\right)+\cos^{-1}\left(\frac{4}{5}\right)$. The rectangular coordinates are $(25\cos\theta,25\sin\theta)$, which is $\left(25\left(\frac{24}{25}\cdot\frac{4}{5}-\left(-\frac{7}{25}\right)\cdot\frac{3}{5}\right),25\left(\left(-\frac{7}{25}\right)\cdot\frac{4}{5}+\frac{24}{25}\cdot\frac{3}{5}\right)\right)=\left(\frac{117}{5},\frac{44}{5}\right)$.

19. Any curve of the form $\theta=k$ is a line, so A is a line, and D is $\tan\theta=1\Rightarrow\theta=\frac{\pi}{4}+n\pi$, which all yield the same line, so D is a line. For B, $r=3\sec\theta=\frac{3}{\cos\theta}\Rightarrow x=r\cos\theta=3$, so B is a line. C is the circle centered at the origin with radius 3, so it is not a line.

20. $\left(\frac{3\pi}{4}-\frac{4\pi}{5}+\frac{5\pi}{6}\right)=135^\circ-144^\circ+150^\circ=141^\circ$

21. $\frac{(3cis72^\circ)^2}{(2cis18^\circ)^3}=\frac{9cis144^\circ}{8cis54^\circ}=\frac{9}{8}cis90^\circ=\frac{9}{8}i$

22. $2011^\circ-9\cdot360^\circ=-1229^\circ$, so that angle is coterminal with the original angle. The answer choices in A, B, and D differ from the original angle by $12{,}066^\circ$, 1980°, and 1260°, none of which is divisible by 360°.

23. $(cis\,x)^2(cis\,y)^3(cis\,z^4)=(cis2x)(cis3y)(cis\,z^4)=cis(2x+3y+z^4)$

24. For A, $x^2+y^2=81\cos^2 t+81\sin^2 t=81\Rightarrow r=\sqrt{x^2+y^2}=9$; likewise for B, so they are both equivalent. For C, $x^2+y^2=(9\sin2t)^2+(9\cos2t)^2=81\sin^2 2t+81\cos^2 2t=81$,

so it is equivalent also. For D, $x^2 + y^2 = \left(9\sqrt{\frac{1-\cos t}{2}}\right)^2 + \left(9\sqrt{\frac{1+\cos t}{2}}\right)^2$

$= 81\left(\frac{1-\cos t}{2}\right) + 81\left(\frac{1+\cos t}{2}\right) = 81$, except that since both coordinates are defined by square roots, this is only the first quadrant portion of the circle, not the entire circle.

25. Mulitplying both sides by r and substituting makes the equation $x^2 + y^2 = 3y - 4x$ $\Rightarrow x^2 + 4x + 4 + y^2 - 3y + \frac{9}{4} = 4 + \frac{9}{4} \Rightarrow (x+2)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{25}{4}$, so the center is at $\left(-2, \frac{3}{2}\right)$.

26. By the last problem, the radius length is $\frac{5}{2} = 2.5$.

27. Cylindrical coordinates are just polar coordinates with a z-coordinate, and they are in the form (r, θ, z), so for this problem, $r = \sqrt{(-2)^2 + 2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ and $\cos\theta = \frac{-2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$ and $\sin\theta = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{3\pi}{4}$. Therefore, the cylindrical coordinates are $\left(2\sqrt{2}, \frac{3\pi}{4}, 5\right)$.

28. The rectangular coordinates are $\left(4\cos\frac{5\pi}{3}, 4\sin\frac{5\pi}{3}, 7\right) = \left(2, -2\sqrt{3}, 7\right)$.

29. Since Santa is at the 60° north latitude, his z-coordinate is $4\sin 60^\circ = 2\sqrt{3}$ and his position in the xy-plane is a distance of $4\cos 60^\circ = 2$ from the origin. Because his position is at $\frac{2\pi}{3}$ radians in the xy-plane, $(x, y) = \left(2\cos\frac{2\pi}{3}, 2\sin\frac{2\pi}{3}\right) = \left(-1, \sqrt{3}\right)$. Therefore, Santa's position's rectangular coordinates are $\left(-1, \sqrt{3}, 2\sqrt{3}\right)$.

30. The angle of inclination is measured down from the z-axis, so we have a right triangle whose hypotenuse is 4, and the side opposite the 15° angle is the radius of the latitude Santa will stay on; call this radius p. Therefore, $p = 4\sin 15^\circ = 4\sin\frac{30^\circ}{2}$

$= 4\sqrt{\dfrac{1-\cos 30^\circ}{2}} = 2\sqrt{2-\sqrt{3}}$, and the circumference is $2\pi p = 4\pi\sqrt{2-\sqrt{3}}$.