

For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. What is the standard form expression of the differential equation $e^x y' - x = y'$?

- A) $y' = -\frac{x}{e^x - 1}$ B) $y' = \frac{x}{e^x - 1}$ C) $y' = -\frac{e^x}{x - 1}$ D) $y' = \frac{e^x}{x - 1}$ E) NOTA

2. What is the solution to the initial value problem $z' - xz = -x$, $z(0) = 4$?

- A) $z = e^{x/2} + 3$ B) $z = 2e^{x^2/2} + 2$ C) $z = 5e^{x^2/2} - 1$ D) $z = 3e^{x^2/2} + 1$ E) NOTA

3. What are the orthogonal trajectories to the family of curves with equations $x^2 + y^2 = c^2$?

- A) $y = kx$ B) $y = kx^2$ C) $y = kx^3$ D) $y = kx^4$ E) NOTA

4. Which of the following is an integrating factor that would yield an exact solution for the equation $2xydx + y^2dy = 0$?

- A) y B) $\frac{1}{y}$ C) $\frac{1}{y^2}$ D) $\frac{1}{y^3}$ E) NOTA

5. Which of the following is the solution set of values of c that makes the set $\{1 - cx, 1 + x, 2 - 6x\}$ linearly dependent?

- A) $c < 1$ B) $c > 1$ C) $-\infty < c < \infty$ D) $c = 3$ E) NOTA

6. A tank initially holds 100 gallons of a brine solution contains 20 pounds of salt. At time $t = 0$, fresh water starts being poured into the tank at a rate of 5 gallons per minute, while the well-stirred, thoroughly-mixed mixture leaves the tank at the same rate. Find the amount of salt in the tank as a function of time t .

- A) $Q = 20e^{-t/20}$ B) $Q = 20e^{-t/30}$ C) $Q = 20e^{-t/40}$ D) $Q = 20e^{-t/50}$ E) NOTA

7. Given that $y' - 2y = 2x$ and $y(0) = 1$, use Euler's Method with a step size of $h = 0.1$ to approximate the value of $y(0.2)$.

- A) 1.40 B) 1.42 C) 1.44 D) 1.46 E) NOTA

8. A particle has acceleration function $a(t) = (t-1)^2$. If the particle is stationary at $t = 1$ and $s(1) = 0$, where s is the position function, what is the particle's position at $t = 3$?

- A) $\frac{4}{3}$ B) 2 C) $\frac{8}{3}$ D) $\frac{10}{3}$ E) NOTA

9. A ball is propelled straight up from the ground with an initial velocity of 256 feet per second in a vacuum with no air resistance. How high will the ball go, in feet, given that acceleration due to gravity is -32 feet per second?

- A) 128 B) 256 C) 512 D) 1024 E) NOTA

10. What constant interest rate is required if an initial deposit placed into an account that accrues interest compounded continuously is to double its value in six years?

- A) $(20\ln 2)\%$ B) $\left(\frac{50\ln 2}{3}\right)\%$ C) $\left(\frac{100\ln 2}{7}\right)\%$ D) $\left(\frac{25\ln 2}{2}\right)\%$ E) NOTA

11. What is the general solution to the differential equation $y'' - y' - 2y = 0$?

- A) $y = c_1e^x + c_2e^{-x}$ B) $y = c_1e^{-x} + c_2e^{-2x}$ C) $y = c_1e^x + c_2e^{2x}$
D) $y = c_1e^{-x} + c_2e^{2x}$ E) NOTA

12. Which of the following is not an ordinary differential equation?

- A) $\sqrt{\frac{dy}{dx}} - y = x$ B) $\frac{d^2y}{dx^2} + y = y^2$ C) $\frac{dx}{dt} + \frac{dt}{dy} = 5ty$ D) $\frac{dy}{dx} - x^2 = xy$ E) NOTA

13. Find the solution to the differential equation $y' - \frac{3}{x}y = x^4y^{1/3}$.

- A) $y = (cx^2 + 2x^5)^{3/2}$ B) $y = \left(cx^2 + \frac{2}{3}x^5\right)^{3/2}$ C) $y = \left(cx^2 + \frac{2}{5}x^5\right)^{3/2}$ D) $y = \left(cx^2 + \frac{2}{7}x^5\right)^{3/2}$
E) NOTA

14. What are the graphs of the family of curves that are solutions to the equation $ydx + xdy = 0$?

- A) parabolas B) hyperbolas C) circles D) ellipses E) NOTA

15. Find the general solution to the differential equation $100\frac{d^2N}{dt^2} - 20\frac{dN}{dt} + N = 0$.

- A) $N = c_1e^{t/10} + c_2te^{t/10}$ B) $N = c_1e^{t/20} + c_2te^{t/20}$ C) $N = c_1e^{t/30} + c_2te^{t/30}$ D) $N = c_1e^{t/40} + c_2te^{t/40}$
 E) NOTA

16. What is the order of the differential equation $ty'' + t^2y' - (\sin t)\sqrt{y} = t^2 - t + 1$?

- A) 1 B) 2 C) 3 D) 4 E) NOTA

17. Given that $\{2, 6, 8\}$ is a complete set of roots for the characteristic equation of an n th-order near homogeneous differential equation in $y(x)$ with real coefficients, determine the associated differential equation.

- A) $y''' - 14y'' + 45y' - 86y = 0$ B) $y''' - 6y'' + 34y' - 72y = 0$ C) $y''' - 18y'' + 58y' - 46y = 0$
 D) $y''' - 16y'' + 76y' - 96y = 0$ E) NOTA

18. Find the general solution to the differential equation $y' = \frac{2y^4 + x^4}{xy^3}$.

- A) $y^4 = x^8 - kx^4$ B) $y^2 = x^8 - x^4$ C) $y^4 = kx^8 - x^2$ D) $y^4 = kx^8 - x^4$ E) NOTA

19. $y = ce^{-\int p(x)dx} + e^{-\int p(x)dx} \int e^{\int p(x)dx} q(x)dx$ is the general solution to which of the following differential equations?

- A) $y' + y = q(x)$ B) $y' + p(x)y = q(x)$ C) $y' + p(x)y = q(x)y^2$
 D) $y' + p(x)y^2 = q(x)y$ E) NOTA

20. Find the general solution to the equation $(x + \sin y)dx + (x \cos y - 2y)dy = 0$.

- A) $x^2 + x \sin y - y = c$ B) $x^2 + x \sin y - y^2 = c$ C) $\frac{x^2}{2} + x \sin y - y^2 = c$
 D) $\frac{x^2}{2} + x \sin y - \frac{y^2}{2} = c$ E) NOTA

21. The population of a certain country is known to increase at any given time at a rate proportional to the number of people living in the country at that time. If after two years the population has doubled, what is the relative growth rate?

- A) $\frac{\ln 2}{4}$ B) $\frac{\ln 2}{2}$ C) $\ln 2$ D) $2\ln 2$ E) NOTA

22. Find the solution to the differential equation $y''' - 6y'' + 11y' - 6y = 0$.

- A) $y = c_1e^x + c_2e^{2x} + c_3e^{3x}$ B) $y = c_1e^x + c_2xe^{2x} + c_3e^{3x}$ C) $y = c_1e^x + c_2xe^{2x} + c_3xe^{3x}$
D) $y = c_1e^x + c_2xe^{2x} + c_3e^{3x} + x$ E) NOTA

23. Find the solution to the differential equation $y' - 3x^2 = x$.

- A) $y = x^3 + x^2 + c$ B) $y = \frac{x^3}{2} + x^2 + c$ C) $y = x^3 + \frac{x^2}{2} + c$ D) $y = \frac{x^3}{2} + \frac{x^2}{2} + c$ E) NOTA

24. Which of the following differential equations is linear?

- A) $y''' - yy'' + y = 5$ B) $(y'')^2 - 5y = 0$ C) $y'' - xy' + y^3 = 0$ D) $y^3y^{(4)} - 4y''' + y = 3$
E) NOTA

25. A metal bar at a temperature of 100°F is placed in a room with a constant temperature of 0°F . If after 20 minutes the temperature of the bar is 50°F , find the time, in minutes, it will take the bar to reach a temperature of 25°F from its initial 100°F .

- A) 30 B) 40 C) 60 D) 80 E) NOTA

26. Find the general solution to the differential equation $y'' - y = x^2$, given that one solution to the equation is $y = -x^2 - 2$ and if two solutions to the differential equation $y'' - y = 0$ are e^x and e^{-x} .

- A) $y = -x^3 - 2 + c_1e^x + c_2e^{-x}$ B) $y = -x^2 - 2 + c_1e^x + c_2e^{-2x}$ C) $y = -x^2 - 2 + c_1e^{2x} + c_2e^{-x}$
D) $y = -x - 2 + c_1e^x + c_2e^{-x}$ E) NOTA

27. Find the product of values c_1 and c_2 such that $y(x) = c_1e^x + c_2e^{-x} + 4\sin x$ satisfies $y(0) = 1$ and $y'(0) = -1$.

- A) -8 B) -6 C) -4 D) -2 E) NOTA

28. The half-life of a new radioactive substance is 1 day. If you begin with 16 grams of the substance, how much of the original substance, in grams, will remain after four days?

- A) 2 B) 1 C) 0.5 D) 0.25 E) NOTA

29. Which of the following differential equations is homogeneous?

- A) $y' = \frac{x^4}{x^2 + y^2}$ B) $y' = \frac{x^4}{x^4 + y^4}$ C) $y' = \frac{x^4}{xy^4}$ D) $y' = \frac{x^4}{y^3}$ E) NOTA

30. Find the solution to the differential equation $y' = \frac{x+1}{y^4+1}$.

- A) $\frac{y^3}{3} + y - \frac{x^2}{2} - x = c$ B) $\frac{y^5}{5} + y - \frac{x^3}{3} - x = c$ C) $\frac{y^5}{5} + y^2 - \frac{x^2}{2} - x = c$
D) $\frac{y^5}{5} + y - \frac{x^2}{2} - x = c$ E) NOTA