

Alpha – Polar Topic Test: National Mu Alpha Theta Convention 2012

1. Which of the following points (r, θ) is NOT located at the same point as the others on a polar graph?
 - a. $(3, -15)$
 - b. $(-3, 15)$
 - c. $(3, 165)$ *change to -3*
 - d. $(-3, -195)$
 - e. None of the Above

2. Which of the following is not a proper match between a given angle and its reference angle?
 - a. $\frac{7\pi}{8}$; reference = $\frac{\pi}{8}$
 - b. $\frac{-7\pi}{8}$; reference = $\frac{\pi}{8}$
 - c. $\frac{-7\pi}{8}$; reference = $\frac{-\pi}{8}$
 - d. $\frac{-9\pi}{8}$; reference = $\frac{\pi}{8}$
 - e. None of the Above

3. Which polar graph looks the most like an infinity symbol?
 - a. Lemniscate
 - b. Limacon
 - c. Rose
 - d. Cardioid
 - e. None of the Above

4. In his book *On Spirals*, this Greek mathematician described a "spiral" whose radius depended on the angle value. Who was this mathematician?
 - a. Aristotle
 - b. Archimedes
 - c. Newton
 - d. Napier
 - e. None of the Above

5. The rectangular point $(20, -15)$ can be converted to polar coordinates. Which of the following is not equivalent to $(20, -15)$?
 - a. $\left(12, \tan^{-1}\left(\frac{3}{4}\right)\right)$
 - b. $\left(25, \tan^{-1}\left(\frac{-3}{4}\right)\right)$
 - c. $\left(12, \tan^{-1}\left(\frac{-3}{4}\right)\right)$
 - d. $\left(25, \tan^{-1}\left(\frac{3}{-4}\right)\right)$ *change to -25*
 - e. None of the Above

6. Conversely, polar coordinates can be converted to rectangular coordinates. Which of the following is equivalent to the polar point $(-4, 15)$ in rectangular coordinates?
 - a. $(\sqrt{6} - \sqrt{2}, \sqrt{6} - \sqrt{2})$
 - b. $(\sqrt{6} + \sqrt{2}, \sqrt{6} - \sqrt{2})$
 - c. $(\sqrt{6} - \sqrt{2}, \sqrt{6} + \sqrt{2})$
 - d. $(\sqrt{6} - \sqrt{2}, -\sqrt{6} + \sqrt{2})$
 - e. None of the Above

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7. Which of the following polar graphs has the greatest rate of change?
- a. $\theta = \frac{-5\pi}{12}$
- b. $\theta = \frac{-7\pi}{12}$
- c. $\theta = \frac{\pi}{12}$
- d. $\theta = \frac{-13\pi}{12}$
- e. None of the Above
8. Which of the following circles has the greatest circumference?
- a. $r = 4$
- b. $r = -\frac{8}{3}$
- c. $r = 8\cos\theta$
- d. $r = -10\sin\theta$
- e. None of the Above
9. Find the equation for a directrix of the following conic section: $r = \frac{8}{2 - 4\cos\theta}$.
- a. $r = \frac{-4}{\cos\theta}$
- b. $r = \frac{4}{\cos\theta}$
- c. $r = \frac{-4}{\sin\theta}$
- d. $r = \frac{8}{\sin\theta}$
- e. None of the Above
10. Find the eccentricity and classify the conic from question #9.
- a. $e = 2$; hyperbola
- b. $e = 2$; ellipse
- c. $e = \frac{1}{2}$; hyperbola
- d. $e = \frac{1}{2}$; ellipse
- e. None of the Above
11. Find a possible equation of an ellipse vertices at $(-2, \pi)$ and $(6, \pi)$.
- a. $r = \frac{4}{2 - \cos\theta}$
- b. $r = \frac{8}{2 - \cos\theta}$
- c. $r = \frac{4}{1 - 2\cos\theta}$
- d. $r = \frac{8}{1 - 2\cos\theta}$
- e. None of the Above
12. Find the product in trigonometric form: $(3 - 3i\sqrt{3})(-4 - 4i)$. $\text{cis}\theta = \cos\theta + i\sin\theta$
- a. $24\text{cis}\frac{-13\pi}{12}$
- b. $4\sqrt{2}\text{cis}\frac{-\pi}{12}$
- c. $24\sqrt{2}\text{cis}\frac{-13\pi}{12}$
- d. $4\sqrt{2}\text{cis}\frac{13\pi}{12}$
- e. None of the Above

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13. $(1-i)^{2012} = ?$

- a. -2^{1006}
- b. $-2^{1006} + 2^{1006}i$
- c. 2^{1006}

- d. $-2^{1006} - 2^{1006}i$
- e. None of the Above

14. The terminal side of angle $\frac{-2012\pi}{3}$ is collinear with which of the following lines?

- a. $\frac{-2\pi}{3} = \theta$
- b. $\frac{-\pi}{3} = \theta$
- c. $\frac{-2\pi}{3} = r$

- d. $\frac{-2}{3} = r$
- e. None of the Above

15. Which is the best classification for the following graph: $r = 2 - 3\cos\theta$.

- a. Rose
- b. Limacon
- c. Lemniscate

- d. Circle
- e. None of the Above

16. How many petals does the following graph have: $r = 3\cos(4\theta)$?

- a. 3
- b. 4
- c. 6

- d. 8
- e. None of the Above

17. Which of the following lines would have an undefined slope when converted to rectangular coordinates?

- a. $\theta = 0$
- b. $r\cos\theta = 0$
- c. $r\sin\theta = 1$

- d. $\theta = -\frac{\pi}{4}$
- e. None of the Above

18. Find the width of one petal of the following graph in radians:

$$r = 4\cos(3\theta)$$

- a. $\frac{\pi}{2}$
- b. $\frac{\pi}{3}$
- c. $\frac{\pi}{4}$

- d. $\frac{\pi}{6}$
- e. None of the Above

Alpha - Polar Topic Test: National Mu Alpha Theta Convention 2012

19. Convert this rectangular equation to polar form: $y^2 = 3x$

a. $r = 3 \tan \theta \csc \theta$

b. $r = \frac{\cot \theta \csc \theta}{3}$

c. $r = 3 \cos \theta \csc \theta$

d. $r = 3 \cot \theta \csc \theta$

e. None of the Above

20. Convert this polar equation to rectangular form: $r = \sin \theta$

a. $x^2 + y^2 - y = 0$

b. $x^2 + y^2 - x = 0$

c. $x^2 + y = 0$

d. $x^2 + y^2 - 1 = 0$

e. None of the Above

21. Find the intersection of the two polar curves: $r = 1 + \cos \theta$ and $r = 1 - \cos \theta$. State the solution in the form (r, θ) where k is an integer.

a. $(1, \frac{-\pi}{2} + \pi k)$

b. $(1, \frac{\pi}{4} + \pi k)$

c. $(-1, \frac{\pi}{4} + \pi k)$

d. $(1, \pi k)$

e. None of the Above

22. Which of the following vectors has a terminal point on the Cartesian plane which is equivalent to the following polar coordinates: $(-3, -120^\circ)$?

a. $\frac{3}{2}i + \frac{3}{2}\sqrt{3}j$

b. $-\frac{3}{2}i + \frac{3}{2}\sqrt{3}j$

c. $\frac{3}{2}i - \frac{3}{2}\sqrt{3}j$

d. $3i + \sqrt{3}j$

e. None of the Above

23. Find the area of the triangle formed by the polar coordinates:

$(2, \frac{17\pi}{3}), (-5, -\frac{40\pi}{3}), (1, -\frac{28\pi}{3})$

a. 5

b. 10

c. $5\sqrt{3}$

d. $10\sqrt{3}$

e. None of the Above

Handwritten notes for Q23:
 $\frac{760}{1080} = \frac{19}{27}$
 120°

24. If ϕ has a value of the tangent of the acute angle formed between the x-axis and a given vector, find ϕ of the angle formed by the vector between the two polar points:

$(13, 156^\circ), (-12, -96^\circ)$


a. 1

b. $\sqrt{3}$

c. $\sqrt{3}/3$

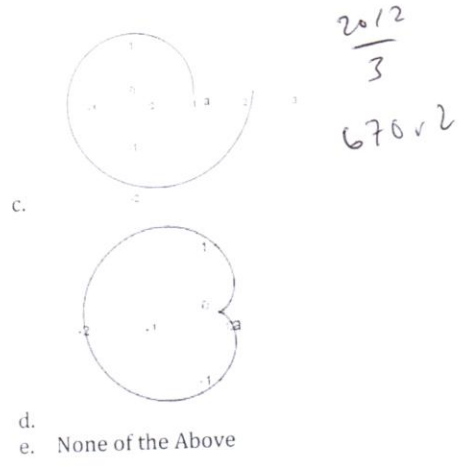
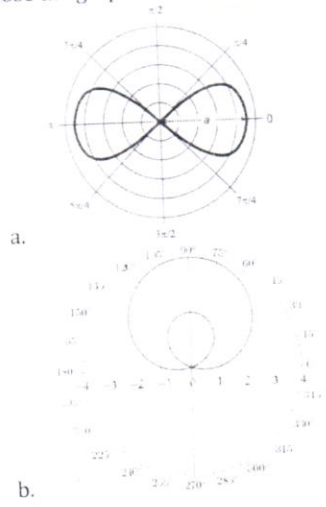
d. 0

e. None of the Above

Handwritten notes for Q24:
 120° 
 $-12 \cos 120^\circ, -12 \sin 120^\circ$
 $(6, -6\sqrt{3})$

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25. Choose the graph that most resembles a lemniscate:



26. Everett is playing a game of Polar Battleship where the ships must lie along a single angle measure or a single radius. The lengths of the ships given in the game are 3 units, 4 units, and 5 units in length. (Disregard the need for a ship to be "linear") One of Everett's ships are located at $(2, \frac{\pi}{3}), (-2, \frac{\pi}{3}), (0, \frac{\pi}{3})$. Which of the following moves by his opponent would be a "hit"?

- a. $(-2, -\frac{2012\pi}{3})$ ~~✗~~
- b. $(\frac{2}{3}, -\frac{2012\pi}{3})$
- c. $(-2, 2012\pi)$ ~~✗~~

- d. $(-2, -2012\pi)$ ~~✗~~
- e. None of the Above

27. Simplify: $(2 - 2i)^7 (1 + i)^6$. Give your answer in trigonometric form.

- a. $(2)^{\frac{27}{2}} \left[\cos -\frac{\pi}{4} + i \sin -\frac{\pi}{4} \right]$
- b. $(2)^{\frac{13}{2}} \left[\cos -\frac{\pi}{4} + i \sin -\frac{\pi}{4} \right]$
- c. $(2)^{\frac{27}{2}} \left[\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right]$
- d. $(2)^{\frac{13}{2}} \left[\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right]$
- e. None of the Above



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28. Find the minimum value of the radius of the following polar equation:

$$r^2 = 16 \sin \theta - 12 \cos \theta$$

- a. 2
- b. $2\sqrt{7}$
- c. 14
- d. $4\sqrt{7}$
- e. None of the Above

29. It is known that Rene Descartes was the first one to develop the plane for rectangular coordinates and that Isaac Newton developed ten different coordinate systems, one of which was the polar system. But which mathematician was the one to coin the terms "pole" and "polar axis"?

- a. G. Leibniz
- b. J. Bernoulli
- c. L. Euler
- d. Archimedes
- e. None of the Above

30. This polar graph is a curve formed by tracing a point on the circumference of a circle that is rolling around a fixed circle with equal radius.

- a. Cardioid
- b. Rose
- c. Line
- d. Lemniscate
- e. None of the Above